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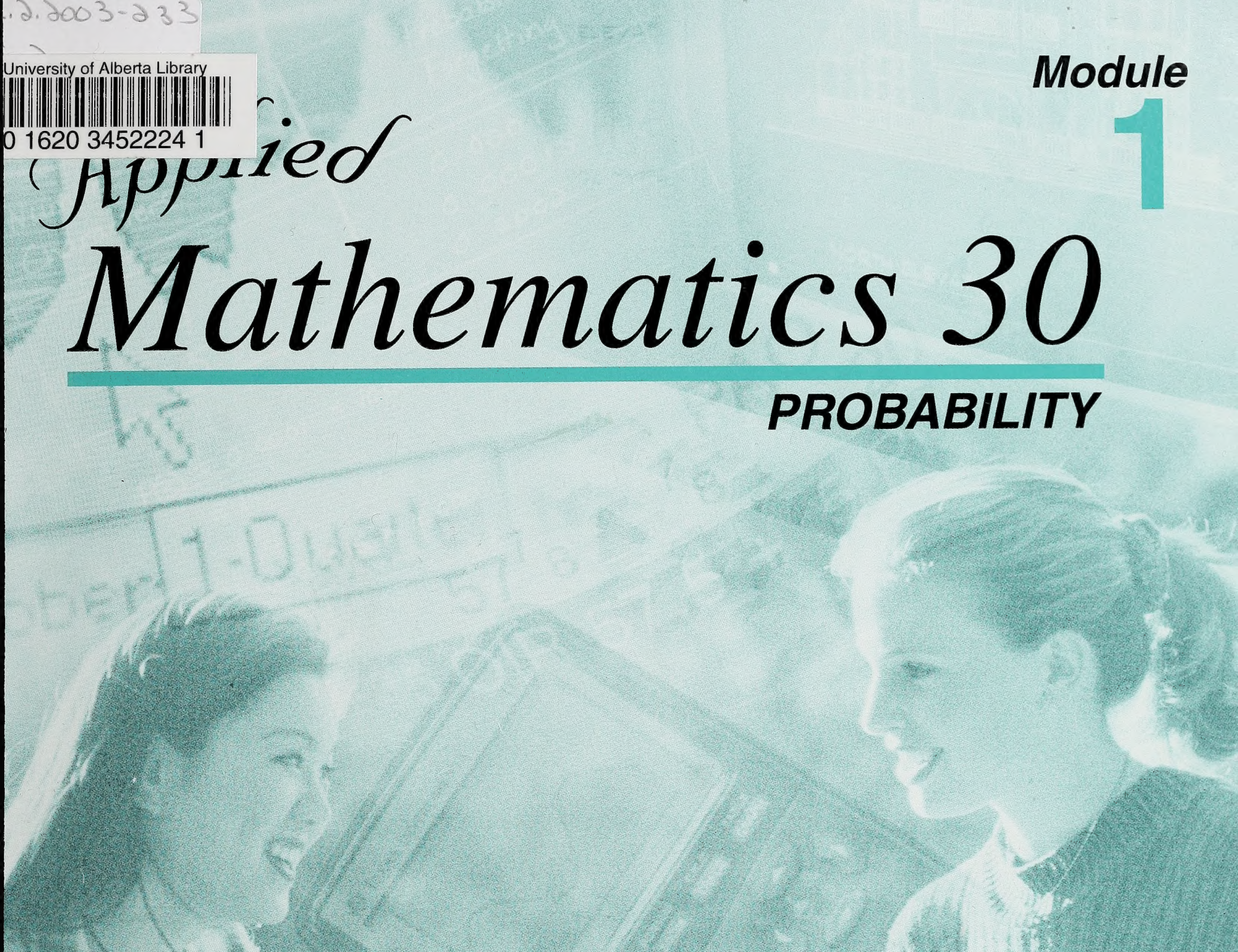
Module
1

Applied Mathematics 30

PROBABILITY



Learning Technologies Branch

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Module

1

Applied Mathematics 30

PROBABILITY

Learning
Technologies
Branch

Alberta
LEARNING

Applied Mathematics 30
Module 1: Probability
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2155-6

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome

Applied Mathematics 30

- Module 1: Probability
- Module 2: Matrices
- Module 3: Statistics
- Module 4: Personal Finance
- Module 5: Sinusoidal Data
- Module 6: Patterns
- Module 7: Vectors

Welcome to Module 1.
We hope you'll enjoy
your study of
Probability.

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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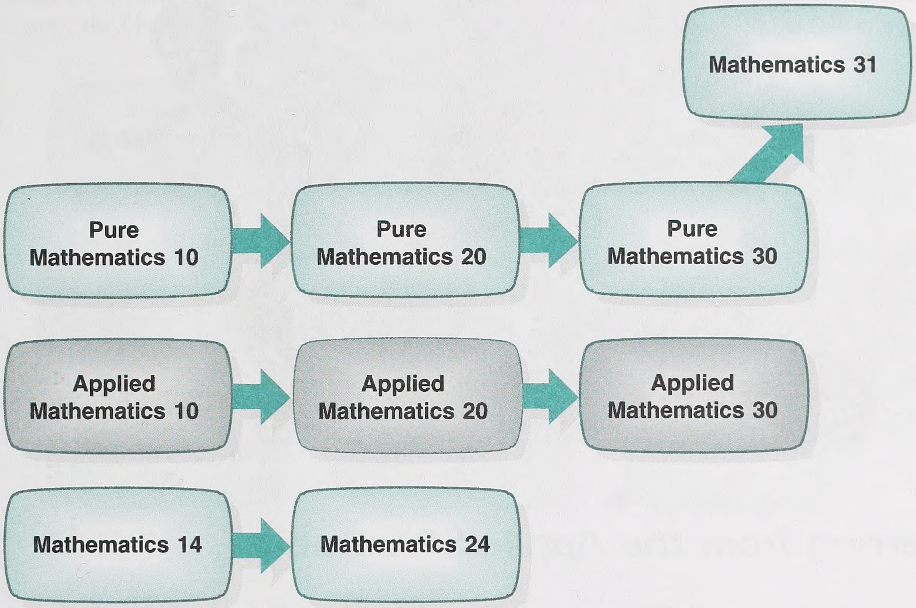
Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading "New Senior High School Mathematics Update/Post-Secondary Studies Update" at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

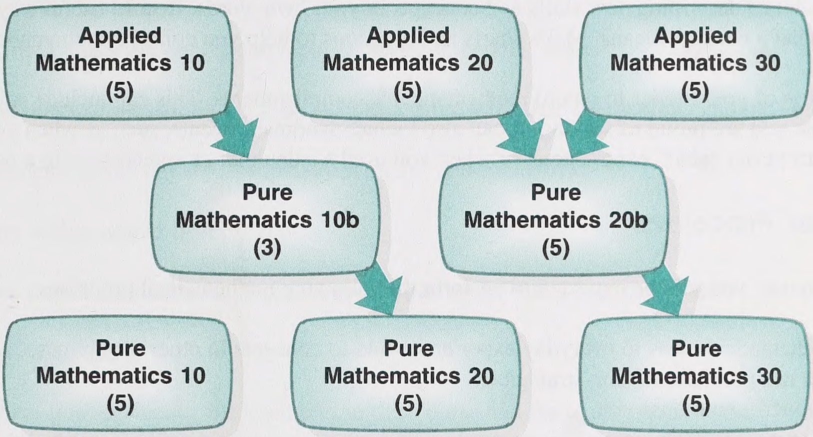
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectations • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

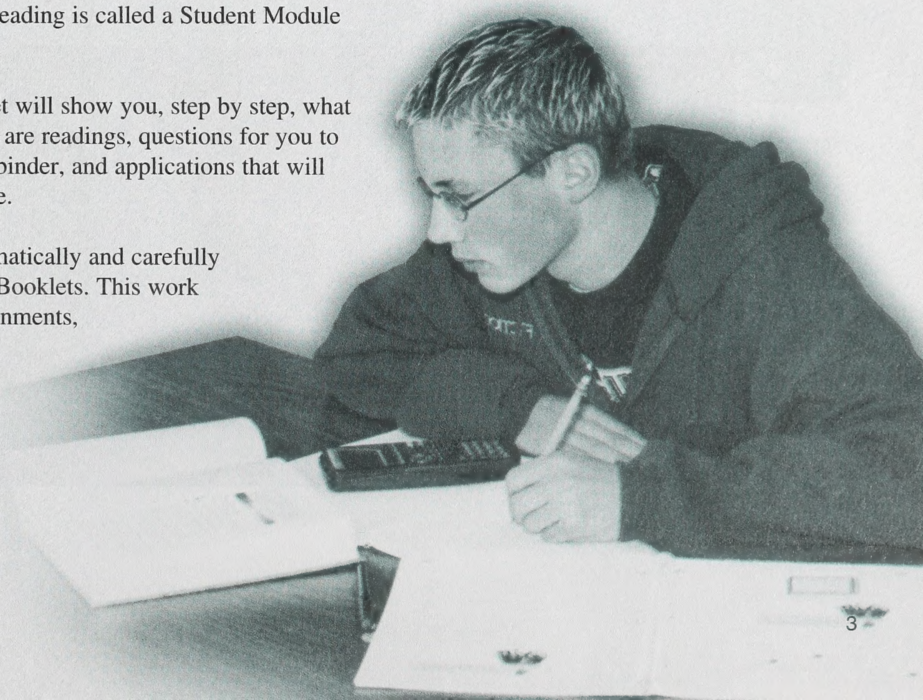


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.
- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83 TI-83 Plus TI-86 TI-89 TI-92* TI-92 Plus	EL-9600C EL-9600* EL-9200* EL-9300*	Algebra FX 2.0 CFX-9850 GA-Plus* CFX-9850 G* CFX-9800 G* FX-9700 series*	HP 39g †

*no longer commercially available

† The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

Refer to the textbook or the Project Book.

Work with a computer.

Refer to the Applied Mathematics 30 CD.

Contact your teacher for additional information.

Explore the Internet.

Complete specified questions in the Assignment Booklet.

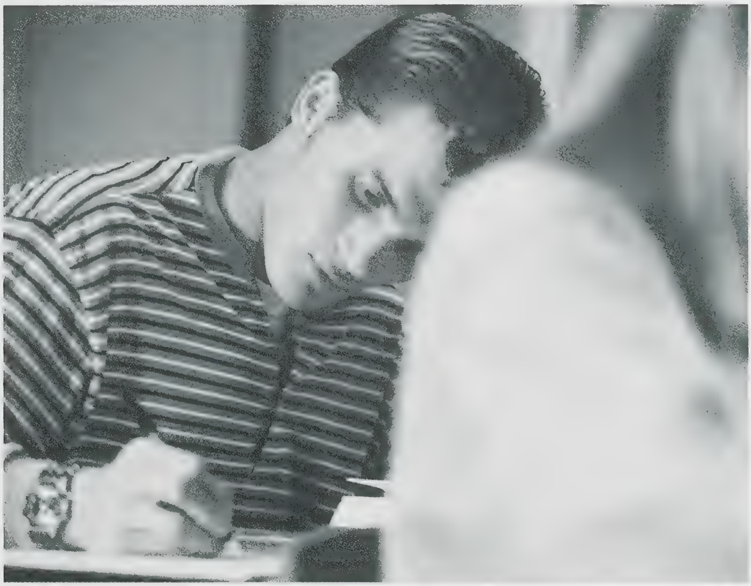
Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.



Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

If you have ever used an automated teller machine (ATM) to make a withdrawal or deposit or to pay bills or if you have ever used a debit card to make a purchase, you are familiar with entering a PIN (personal identification number) to access your accounts. A PIN consists of at least four digits that are easy for you to remember and difficult for others to guess. When selecting a PIN, always try to avoid numbers like your telephone number or your birthday.

Accessing an e-mail account also requires a password. The difference between a password for an e-mail account and a PIN is that the password for the e-mail account can consist of letters (a word or name) or a combination of letters and numbers. Can you imagine the number of possible passwords when letters and numbers can be used? What is the chance of someone breaching your privacy by accessing your password within three tries?

In this module, you will explore passwords and password security as part of the module project. You will also investigate experimental and theoretical probability and the number of ways a series of tasks in probability problems can be done; and you will focus on procedures for determining probabilities of given events. You will then classify events and use them to help you decide how to approach probability problems, particularly those involving key words, like *and*, *or*, *not*, *at least*, and *at most*.

Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

Assignment Booklet 1A	
Activities 1 to 3 Assignment	**90 marks**
Assignment Booklet 1B	
Activities 4 and 5 Assignment	**20 marks**
Module Review Assignment	**50 marks**
Module Project	**40 marks**
TOTAL	**200 marks**

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT

Voice-Mail Security

Beginning the Project

Your teacher may not require you to complete all the projects provided in this Applied Mathematics 30 course. Contact your teacher and check whether you need to complete the Module Project, Voice-Mail Security, as part of your assessment.

Many professionals, office workers, people in the trades, and busy families use voice mail to ensure that important messages left by clients, colleagues, friends, and family will reach them in a secure and timely fashion. To access their voice messages, users must enter a password by pressing combinations of keys on a touch-tone telephone keypad. To ensure privacy, the password should be easy for the user to remember and difficult for others to guess.

The project for Module 1 is Voice-Mail Security. This project involves examining two proposals for assigning passwords for use with a voice-mail system offered to subscribers of a local telephone company. As part of your project you will suggest alternatives or modifications to improve these proposals.

To begin your project, turn to page 2 of the textbook and read "Voice-Mail Security," answering the questions posed near the bottom of the page. Store your reponses in the project section of your mathematics binder.

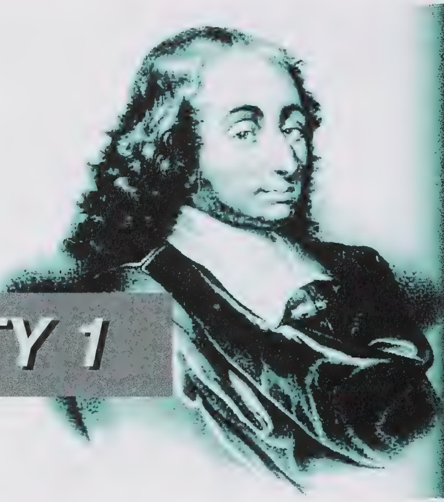
After you have recorded some of your initial ideas regarding the project, begin researching some of the things you need to know about voice mail and voice-mail security.

To start, you may find it useful to visit Addison-Wesley's website described on page 3 of the textbook. This website has links to several other websites you may find helpful in researching voice-mail security.

As you work through Activities 1 to 5, continue to research ideas for voice-mail security by using the Internet, by reading the description of your telephone company's current voice-mail system in the telephone directory, by talking to a representative of your local telephone company, or by reading promotional brochures. Concepts presented in this module, such as the Fundamental Counting Principle and probability techniques, will be useful in completing this project.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner or a family member. Remember, the work on the project you submit must be your own.

ACTIVITY 1



Blaise Pascal (1623–1662)

Pierre de Fermat (1601–1665)

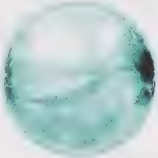
Experimental and Theoretical Probability

Probability theory was developed from the analysis of games of chance. In the middle of the seventeenth century, a habitual gambler, the Chevalier de Méré, asked a mathematician by the name of Blaise Pascal to help him solve the following problem:

> If a game of chance consisting of a certain number of rounds must be cancelled before it is completed, how should the money bet on the entire game be divided among the players if the number of rounds each player has won is known?

By solving this problem, Pascal, together with another mathematician by the name of Pierre de Fermat, laid the basis of probability theory.

Today, probability has applications that extend well beyond gaming. It is used in such areas as medicine, statistics, quality control, meteorology, economics, and insurance to name just a few.



If you have access to the Internet, you can find out more about Blaise Pascal and his contribution to probability at the following website:

http://www.maths.tcd.ie/pub/HistMath/People/Pascal/RouseBall/RB_Pascal.htm

In this activity, you will estimate and calculate probabilities. You are already familiar with probabilities in a variety of everyday contexts. Did you listen to the weather forecast this morning? What is the probability of precipitation? What is meant, for example, by the statement, "The probability of rain today is 60%"? Do you agree that if the same probability of rain existed for 10 days straight, you would expect rain on 6 of those 10 days?

$$\begin{aligned} P(\text{rain}) &= 60\% \\ &= \frac{6}{10} \\ &= 0.6 \end{aligned}$$

"*P*(rain)" is read as *the probability of rain.*

If the probability of rain was 0% over a ten-day period, you would expect no rain over these ten days. Likewise, if the probability of rain was 100% over a ten-day period, you would expect rain on each of these ten days.

$$\begin{aligned} P(\text{rain}) &= 0\% \\ &= \frac{0}{10} \\ &= 0 \end{aligned} \qquad \begin{aligned} P(\text{rain}) &= 100\% \\ &= \frac{10}{10} \\ &= 1 \end{aligned}$$

Probabilities range between 0% and 100% and may be written as percents, common fractions, or decimal fractions.

For a particular outcome of an experiment (also known as an **event**), E, $0\% \le P(E) \le 100\%$ or, equivalently, $0 \le P(E) \le 1$.

You will know you have made a mistake in calculating a probability if your answer is larger than 1 (or 100%) or less than 0 (or 0%).

Now, you will explore how to calculate probabilities. Turn to page 4 of the textbook and read the introductory paragraphs of Tutorial 1.1, "Experimental and Theoretical Probability."

1. Complete exercises 1 to 3 of "Investigation: Finding Experimental and Theoretical Probabilities" on pages 4 and 5 of the textbook. For exercise 2, if you are unable to collect data from other groups, use the Coin Toss data on pages 148 and 149 of the Appendix.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 63–65.

In Part A of the preceding investigation, you estimated the probability of obtaining exactly two heads when three coins are tossed. These estimates are examples of **experimental probability**. In Part B, you used a tree diagram to determine the relative frequency of obtaining two heads when three coins are tossed. You assumed that the probability of obtaining a head in each toss was 0.5. The answer to exercise 3 of the investigation is an example of **theoretical probability**.

To explore more about experimental and theoretical probability, turn to page 5 of the textbook and read the paragraphs following the investigation.

2. Classify each of the following probabilities as either experimental or theoretical. Justify your answer in each case.

   a. The probability of obtaining a 1 on the toss of a fair die is $\frac{1}{6}$.

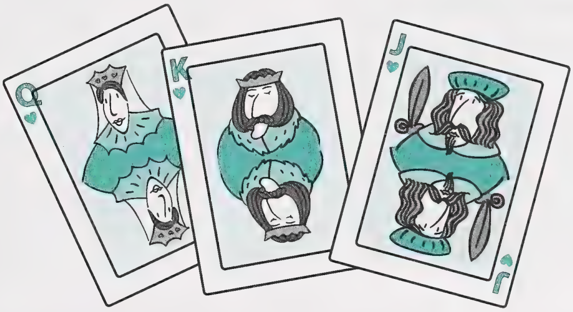
   b. An insurance company has monitored the number of claims of newly licensed drivers over a five-year period. On average, during each of those years there were 50 claims for every 300 drivers. The probability that a newly licensed driver will make a claim in any given year is $\frac{1}{6}$.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 65.

Often, you can use your graphing calculator to simulate an experiment. Consider the following example.

Example

In a standard deck of 52 cards, there are four suits: clubs, diamonds, hearts, and spades. The king, queen, and jack of each suit are called face cards. The deck is shuffled, and one card is drawn at random.



a. What is the theoretical probability of drawing a face card? Express your answer as a fraction and as a percent rounded to the nearest tenth.

b. Simulate the drawing of a single card from a shuffled deck 100 times and record the number of times a face card appears.

c. From your simulation, what is the experimental probability of drawing a face card? Compare this probability with the theoretical probability.

Solution

a. There are three face cards in each suit; therefore, there are $4 \times 3 = 12$ face cards altogether.

$$\begin{aligned} \therefore\ P(\text{face card}) &= \frac{\text{Number of face cards}}{\text{Total number of cards}} \\ &= \frac{12}{52} \\ &= \frac{3}{13} \text{ or about } 23.1\% \end{aligned}$$

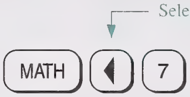
$$\frac{3}{13} \times 100\% \doteq 23.1\%$$

The theoretical probability of drawing a face card is $\frac{3}{13}$, or approximately 23.1%.

b. Use the Random Binary Number feature on your graphing calculator.

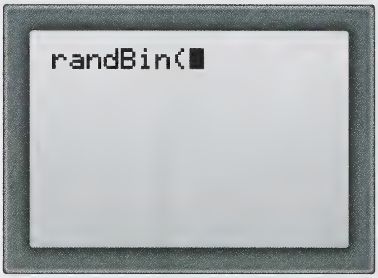
> **Note:** Because this simulation involves the graphing calculator choosing the outcomes randomly, the results you obtain while working through this example will vary slightly.

Step 1: Select the Random Binary Number feature.



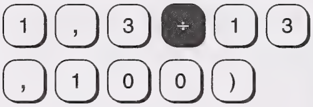
Select the PRB menu.

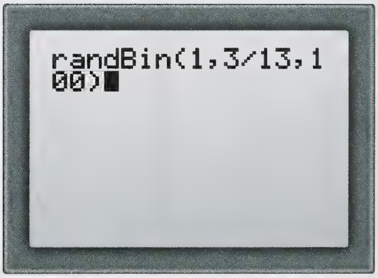
(7:randBin()



```
randBin(■
```

Step 2: Enter the information for the simulation.

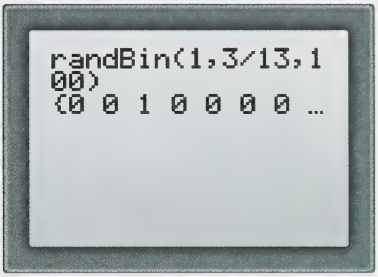




```
randBin(1,3/13,1
00)■
```

- randBin(1, 3/13, 100)
  This indicates that when a face card is drawn, a 1 will appear in the outcomes; otherwise, a 0 will appear.
- randBin(1, 3/13, 100)
  This is the theoretical probability of drawing a face card.
- randBin(1, 3/13, 100)
  This is the number of trials in the simulation.

Step 3: Press ENTER to obtain the outcomes of the simulation.

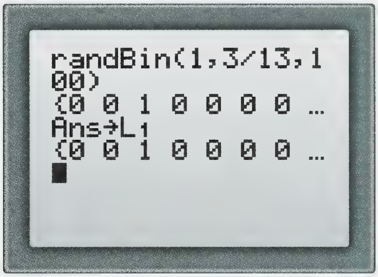


```
randBin(1,3/13,1
00)
{0 0 1 0 0 0 0 …
```

Notice that only one face card was drawn in the first seven trials. To determine the total number of face cards drawn in all 100 trials without having to scroll through the outcomes, store the outcomes as a list first.

2nd [ANS] STO➡

2nd [L1] ENTER

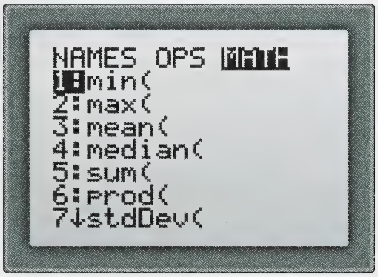


```
randBin(1,3/13,1
00)
{0 0 1 0 0 0 0 …
Ans→L1
{0 0 1 0 0 0 0 …
█
```

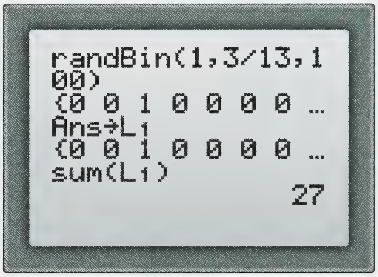
Then total the list.

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L1]) ENTER



```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
```



```
randBin(1,3/13,1
00)
{0 0 1 0 0 0 0 …
Ans→L1
{0 0 1 0 0 0 0 …
sum(L1)
                  27
```

In this simulation, a 1 occurred 27 out of 100 times.

c. The experimental probability of drawing a face card is $\frac{27}{100}$ or 27%. This result is close to the theoretical probability.

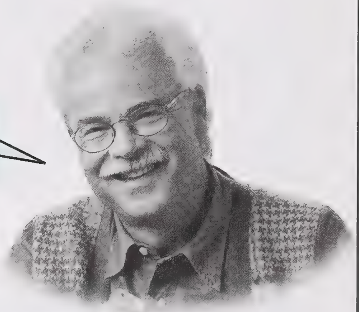
When you worked through the preceding example, your list and experimental probability most likely differed slightly from those in the example. Again, these differences occur because the lists are randomly generated, just as you would expect different results when you draw cards randomly from a deck of cards. Of course, the more trials you conduct, the closer the experimental probability will be to the theoretical probability.

Also, in the preceding example, you used a variation of the formula for theoretical probability of an event, *A*. For an experiment that has *n* equally likely outcomes, *r* of which are favourable to event *A*,

$$\text{Probability of event } A = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(A) = \frac{r}{n}$$

Next, you will practise finding experimental and theoretical probabilities.



Turn to pages 6 to 9 of the textbook and work through "Example 1: Determine the Probability of Rolling a 5," "Example 2: Determine the Probability of a Pathway," and "Example 3: Determine the Probability of Drawing a Particular Card."

3. Answer the following on pages 9 to 12 of the textbook.

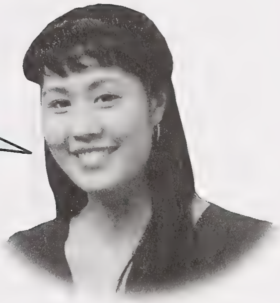
a. exercises 2 and 3 of "Discussing the Ideas"
b. exercises 1 to 10 of "Exercises: Checking Your Skills"

Note: For exercise 4, some graphing calculators may not be able to simulate all 300 trials at once. If this is the case with your calculator, simulate 100 trials three times and add the three simulations together.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 65–74.

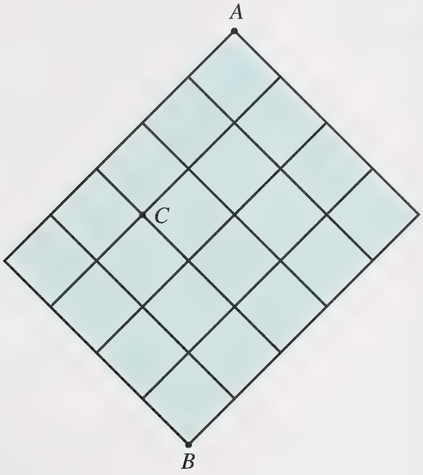
Turn to pages 1 and 2 of Assignment Booklet 1A and answer questions 1 to 3.

One type of probability problem you encountered in the previous exercises involved pathways. Here is another example that will help you learn to solve these problems.



Example

Use the diagram given to answer the following.



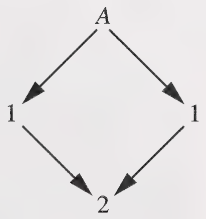
a. Determine all the possible ways to get from point A to point B if each move must be downward and either to the right or left.

b. If a path from A to B is selected, what is the probability that it will pass through point C?

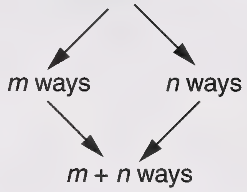
Solution

a. Start at point A and determine the number of ways of reaching each node, located at the intersections of the gridlines.

Consider the first square.

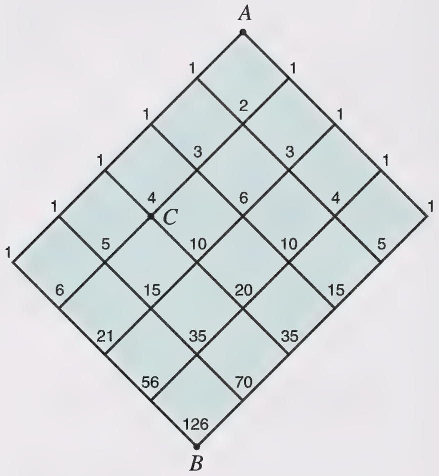


There is one way of moving 1 unit to the lower left. Likewise, there is one way of moving 1 unit to the lower right. Therefore, there are $1+1=2$ ways of completing the first square. The following principle is continued downward.



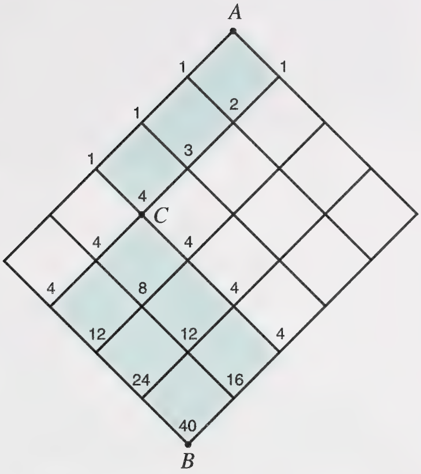
For a given square on the grid, if there are *m* ways of reaching the node on the left and *n* ways of reaching the node on the right, there are $m+n$ ways of reaching the node that completes the square.

Write the number of ways of arriving at each node, continuing the pattern until you reach point *B*.



There are 126 ways of getting from point *A* to point *B*.

b. First, find the number of paths from *A* to *B* through point *C*. In part a., you determined that there were 4 ways of reaching point *C*. Continue from point *C* until you reach point *B*.

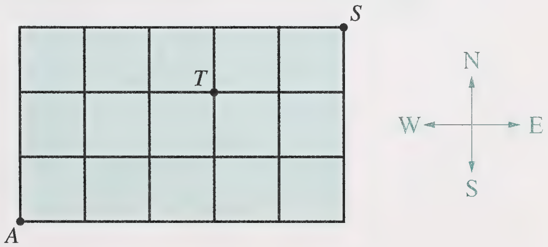


There are 40 ways of reaching point *B* through point *C*. Therefore, the probability that the path from point *A* to point *B*, chosen at random, passes through point *C* is

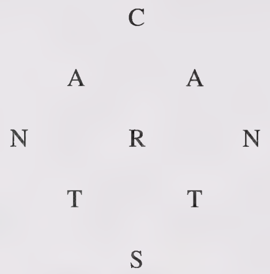
$$
\begin{aligned}
P(A \text{ to } B, \text{ through } C) &= \frac{40}{126} \\
&= \frac{20}{53} \text{ or about } 37.7\%
\end{aligned}
$$

Now it's your turn to practise these skills!

4. Antonella's school, *S*, lies 5 blocks east and 3 blocks north of her home, *A*. If she walks to school only travelling north or east and chooses her route at random, what is the probability that her route will take her past the town hall, *T*? Express the probability as a fraction and a percent.

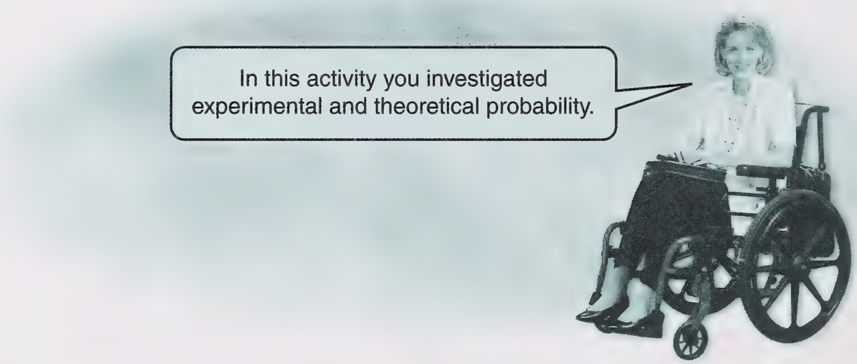


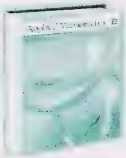
5. Starting at the letter *C*, move randomly downward to the left or to the right. What is the probability that the path you select spells CARTS? Express the probability as a fraction.



Compare your responses with the suggested answers in the Appendix, Activity 1, pages 74–75.

Looking Back


In this activity you investigated experimental and theoretical probability.



6. Turn to page 12 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 75.

ACTIVITY 2

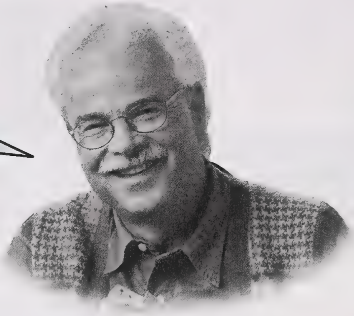
Generating and Using Sample Spaces

If you play board games that involve moving your game piece the number of squares determined by the toss of two dice, you know from experience that certain sums are more probable than others. Obtaining a sum of 7 is the most likely, whereas obtaining a sum of 2 or a sum of 12 are the least likely.

In Activity 1 you determined the probability of a particular event by comparing the number of outcomes favourable to the event with the total number of possible outcomes. The set of all possible outcomes of an experiment is the sample space.

Knowing the number of outcomes in sample spaces is essential when determining theoretical probabilities. One method is to list all outcomes and count them. But how will you know if you have listed them all? In this activity, you will explore systematic approaches to listing outcomes.

These systematic approaches will assist you when determining all possible outcomes of a particular experiment, such as the toss of two dice. You will begin by using tables and grids.

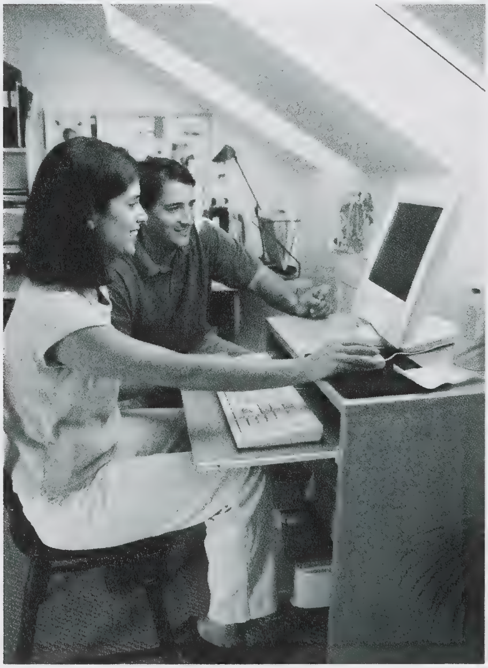


Turn to page 13 of the textbook and read the introductory paragraphs of Tutorial 1.2, “Generating and Using Sample Spaces.”

1. Complete exercises 1 to 8 of “Investigation: Rolling Two Dice” on pages 13 and 14 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 76–82.

To see probability in action, open the explorer *Hockey Shootout* on the Applied Mathematics 30 CD.



In the preceding investigation, you considered the releationship between the probability that an event will occur and the probability that the event will not occur. Here is another example.

Example

The weather forecast states that the probability of rain today is 30%. What is the probability that it will not rain today?

Solution

Let R be the event described by the weather forecast.

$$\begin{aligned} P(R) &= \text{probability of rain} \\ &= 30\% \text{ or } 0.30 \end{aligned}$$

Let $\overline{R}$ be the event described by "no rain today."

$$\begin{aligned} P(\overline{R}) &= \text{probability of no rain} \\ &= 100\% - P(R) \\ &= 100\% - 30\% \\ &= 70\% \text{ or } 0.70 \end{aligned}$$

The probability that it will not rain is 70%.

In the example, you worked with the **complement of an event**; event $\overline{R}$ is the complement of event R. For any two complementary events, R and $\overline{R}$,

$$P(R)+P(\overline{R})=100\% \text{ or } P(R)+P(\overline{R})=1$$

As you will discover, it is often easier to determine $P(\overline{R})$ and then use it to calculate $P(R)$. To find out more about complementary events, read the information following the investigation on pages 14 and 15 of the textbook.

2. Answer exercises 1, 2, and 3.a. of "Exercises: Checking Your Skills" on page 17.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 82.

At the beginning of this activity, you generated sample spaces using tables and grids. Another method of generating sample spaces is using a tree diagram.

Example

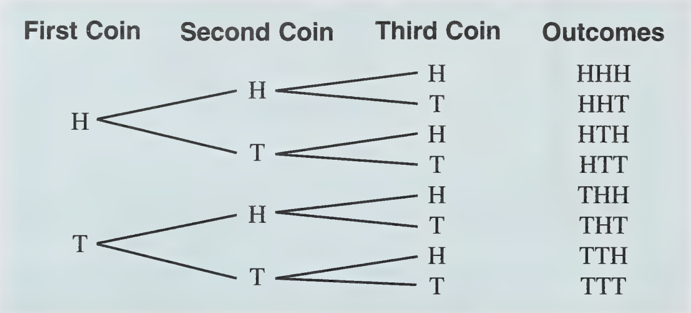
Three coins are tossed.

a. Use a tree diagram to determine every possible outcome.

b. What is the probability of getting 2 heads and 1 tail?

Solution

a. Each coin can turn up heads (H) or tails (T).



There are 8 outcomes in the sample space:

$$\{(\text{H, H, H}), (\text{H, H, T}), (\text{H, T, H}), (\text{H, T, T}), (\text{T, H, H}), (\text{T, H, T}), (\text{T, T, H}), (\text{T, T, T})\}$$

Note how each outcome can be written as an ordered triplet: (first coin, second coin, third coin).

b. Let E be the event of getting 2 heads and 1 tail.

There are 3 outcomes in the event: (H, H, T), (H, T, H), and (T, H, H).

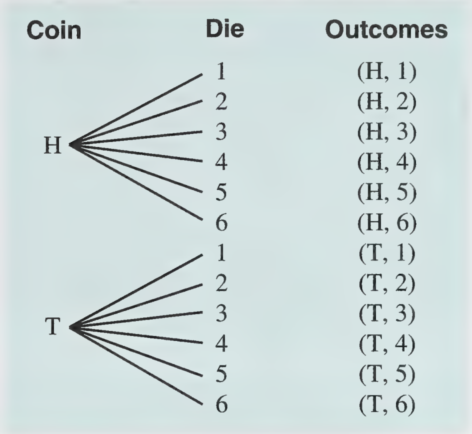
$$\therefore\ P(E) = \frac{3}{8}$$

Example

A coin is tossed and a die is cast. What is the probability of getting a head and rolling a 3 on the die?

Solution

Draw a tree diagram to determine all possible outcomes.



There are 12 possible outcomes in the sample space.

Let *E* be the event of getting a head (H) and rolling a 3.

There is 1 outcome in the event: (H, 3).

$$\therefore\ P(E) = \frac{1}{12}$$

Now, it is time for you to practise finding events, sample spaces, and probabilities on your own.

Turn to pages 15 and 16 of the textbook and work through "Example: Tossing 3 Coins."

3. Answer the following on pages 16 to 19 of the textbook.

  a. exercises 1 and 3 of "Discussing the Ideas"
  b. exercises 4, 5, 7, and 8, of "Exercises: Checking Your Skills"
  c. exercise 10 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 83–90.

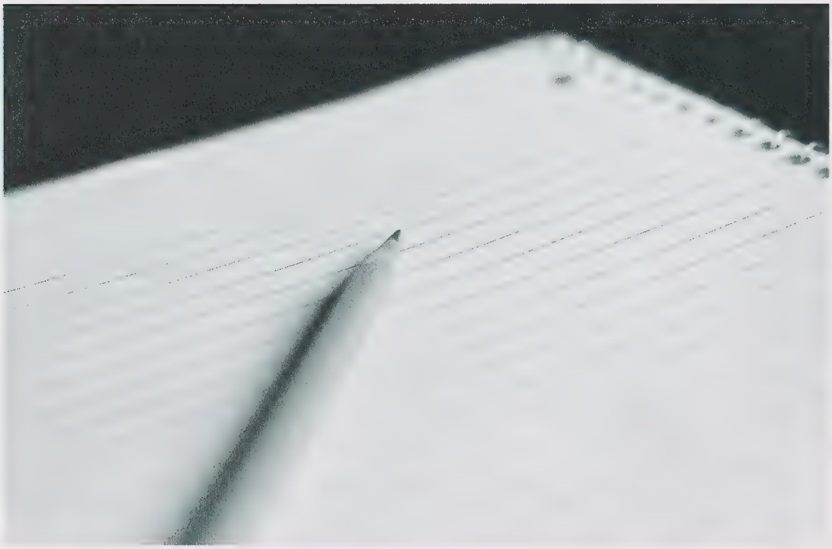
Turn to pages 2 and 3 of Assignment Booklet 1A and answer questions 4 and 5.

Looking Back

In this activity, you constructed sample spaces for events, determined the number of outcomes for events by selecting favourable outcomes from the sample spaces, and calculated probabilities for events and their complements.

4. Turn to page 19 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, pages 90–91.



ACTIVITY 3

The Fundamental Counting Principle

Before the year 2000, there was just one area code in Alberta. Because of an increased number of households and businesses with telephones, cell phones, Internet connections, fax machines, and so on, the telephone companies were running out of telephone numbers. So, the province was allocated two area codes: 780 for use in Northern Alberta and 403 for use in Southern Alberta.

Have you wondered how many seven-digit telephone numbers are possible, how many standard licence plates consisting of three letters followed by three digits (like those in Saskatchewan and Alberta) can be made, or how many different postal codes can be assigned? These questions can be answered using the Fundamental Counting Principle.

In this activity, you will explore the Fundamental Counting Principle. In particular, you will investigate how you can use it to count outcomes in sample spaces and to answer probability questions.

To see how the Fundamental Counting Principle relates to routing problems, view the segment *Postal Routes as a Pathway Problem* on the Applied Mathematics 30 CD.

Turn to page 20 of the textbook and read the introductory paragraph of Tutorial 1.3, “The Fundamental Counting Principle.”

1. Complete exercises 1 to 4 of “Investigation: Counting Without Counting” on page 20 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 92–94.

The previous investigation introduced you to the Fundamental Counting Principle.

FUNDAMENTAL COUNTING PRINCIPLE

If one task can be performed in m different ways and a second task can be performed in n different ways, together, the two tasks can be performed in $m \times n$ different ways.

If you have access to the Internet, you can discover more about the Fundamental Counting Principle and its application at the following website:

http://ppatten.ngc.peachnet.edu/COURSES97_98/count.html

Now, turn to page 21 of the textbook and work through “Example 1: Purchasing a Computer.”

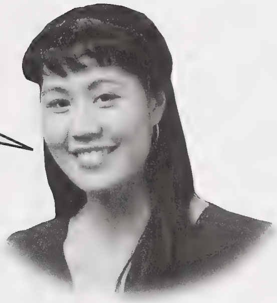
2. Answer the following on page 23 of the textbook.

   a. exercises 1 and 3 of “Discussing the Ideas”
   b. exercises 1, 2, and 3 of “Exercises: Checking Your Skills”

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 94–96.

Turn to page 4 of Assignment Booklet 1A and answer question 6.

Be careful when using the Fundamental Counting Principle to solve problems. Make sure you check for any restrictions on any of the tasks.



Example

A natural number is a counting number. The first three natural numbers are 1, 2, and 3.

a. How many three-digit, natural numbers are there?

b. How many even, three-digit, natural numbers greater than 499 are there?

c. A computer is programmed to generate a random three-digit, natural number. What is the probability that this number is even and greater than 499?

Solution

a. Three-digit, natural numbers lie between 100 and 999 inclusive. To form these numbers, you must select hundreds, tens, and ones digits.

hundreds	tens	ones

The hundreds digit can be any digit from 1 to 9; it cannot be a 0. There are 9 ways to select the hundreds digit.

9		
hundreds	tens	ones

The tens digit can be any digit from 0 to 9. There are 10 ways to select the tens digit.

9	10	
hundreds	tens	ones

The ones digit can be any digit from 0 to 9. There are 10 ways to select the ones digit.

9	10	10
hundreds	tens	ones

Using the Fundamental Counting Principle, there are $9 \times 10 \times 10 = 900$ three-digit, natural numbers.

b. Again, you must select hundreds, tens, and ones digits. However, this time the numbers must be even numbers between 500 and 998 inclusive.

The hundreds digit can be any digit from 5 to 9. There are 5 ways to select the hundreds digit.

5		
hundreds	tens	ones

The tens digit can be any digit from 0 to 9. There are 10 ways to select the tens digit.

5	10	
hundreds	tens	ones

The ones digit must be even. For the three-digit number to be even, it must end in a 0, 2, 4, 6, or 8. There are 5 ways to select the ones digit.

5	10	5
hundreds	tens	ones

Using the Fundamental Counting Principle, there are $5 \times 10 \times 5 = 250$ even, three-digit, natural numbers greater than 499.

c. Let *E* be the event of selecting an even, three-digit, natural number greater than 499. There are 250 outcomes in *E*.

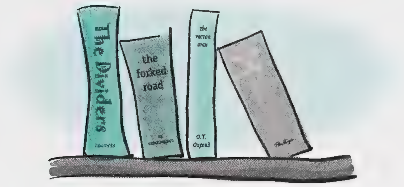
There are 900 three-digit, natural numbers possible. Thus, there are 900 outcomes in the sample space.

$$\begin{aligned} \therefore\ P(E) &= \frac{250}{900} \\ &= \frac{5}{18} \text{ or about } 27.8\% \end{aligned}$$

The probability that the computer will generate an even, three-digit, natural number greater than 499 is $\frac{5}{18}$, or about 27.8%.

Example

Four novels by different authors are placed at random on a shelf. What is the probability that the novels will be arranged with the authors' names in alphabetical order?



Solution

There are four tasks: choosing the first book, choosing the second book, choosing the third book, and choosing the fourth book to place on the shelf.

first book	second book	third book	fourth book

There are 4 ways of selecting the first book.

Once the first book is placed on the shelf, there are only three books left. Therefore, there are 3 ways of selecting the second book.

After the first and second books are placed on the shelf, there are only two books left. Therefore, there are 2 ways of selecting the third book.

After the first three books are placed on the shelf, there is only one book left. Therefore, there is only 1 way of selecting the fourth book.

first book	second book	third book	fourth book
4	3	2	1

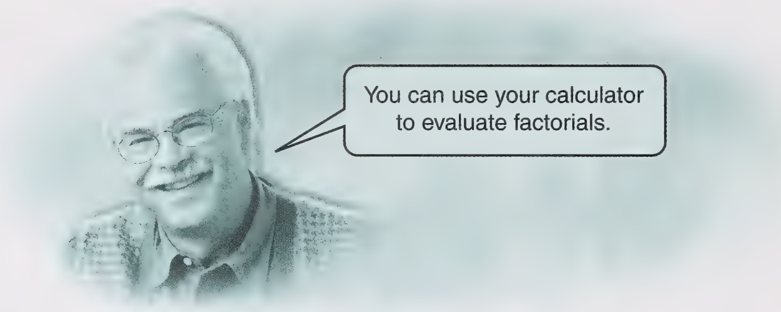
Using the Fundamental Counting Principle, there are $4 \times 3 \times 2 \times 1 = 24$ ways of arranging the four books on the shelf. Only one of these arrangements has the books arranged with the authors' names in alphabetical order. Therefore, the probability that the books are placed on the shelf with the authors' names in alphabetical order is $\frac{1}{24}$, or about 4.2%.

In the preceding example, you calculated the product $4 \times 3 \times 2 \times 1$. Products of this type, where the factors decrease by one, are common in problems involving the Fundamental Counting Principle. For example, the number of ways of arranging nine people in a line for a photograph is

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362\,880$$

There is a way to represent these products, called factorial notation, without having to write out the entire product.

For example, $4 \times 3 \times 2 \times 1$ may be written as 4!. This is read as *four factorial*.


You can use your calculator to evaluate factorials.

Example

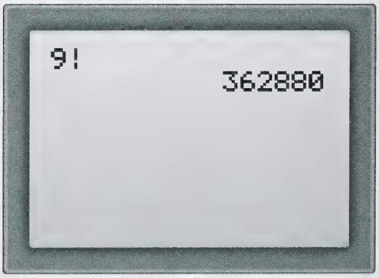
Use your graphing calculator to evaluate 9!.

Remember:
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Solution

Select the PRB menu.

9 MATH ◀ 4 (4:!) ENTER



```
9!
                362880
```

You will find the Factorial feature on your calculator useful. It will help avoid long calculations.

3. Write each of the following as a product; then use the Factorial feature on your calculator to evaluate the product.

   a. 6! **b.** 10! **c.** $(0.5)!$ **d.** $(-2)!$

Compare your responses with the suggested answers in the Appendix, Activity 3, page 96.

The following examples and exercises involve restrictions. Using factorial notation often makes finding solutions simpler.

Turn to pages 21 and 22 of the textbook and work through "Example 2: How Many Digits Are Possible?" and "Example 3: Calculating Probabilities."

4. Answer the following on pages 24 and 25 of the textbook.

   a. exercises 5 to 10 of "Exercises: Checking Your Skills"
   b. exercise 11 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 97–101.

Project: Shelving Systems

There is a very high probability that most people would tell you they could use help in keeping their closets more tidy. There is an apparent problem with finding the proper place to store things or with finding any place for that matter. By applying both effort an design skills, a solution can often be found. You are going to look at designs for organizers, which might be a part of the solution.

Before moving on, you might find the following websites to be useful sources of information regarding building, designing, or buying a closet organizer.

- **http://www.easyclosets.com**
- **http://www.hgtv.com/HGTV/project/0,1158,DEID_project_17881,00.html**
- **http://www.homestore.com/home_improvement/howtoguides/CreativeConstructingBuiltInShelvingUnit.asp?**
- **http://www.ryobitools.com/main/project/pdfdocs/projectl.pdf**

One part of most closet organizers is a set of shelves either with or without drawers. Th Shelving Systems Project on pages 136 to 141 of the Project Book looks at the design and construction of a set of shelves. Turn to page 136 of the Project Book and read “Th Task” and “Background.” Answer the questions posed in “Background” and write you responses in your mathematics binder.

You are going to work with the shelving plans given in the Project Book. You will develop a spreadsheet to itemize the parts and packaging (along with costs) for this shelving system. Along the way, you will practise using volume, area, and costing calculations you have mastered in earlier courses as well as your spreadsheet skills.

There are many costs that are not accounted for in the Project Book presentation. For example, no costs are assigned to cutting the boards, drilling holes, applying the melamine edging, counting and bagging the parts, putting the box together or putting the parts into the box. You will be asked to estimate these costs in one of the questions in Assignment Booklet 1A.

5. Answer the following on pages 138 and 139 of the Project Book.

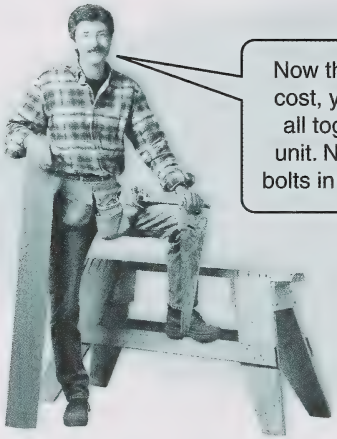
- **a.** exercise 1 of "Getting Started" as if you were building exactly one shelving unit
- **b.** exercise 1 of "Getting Started" as if you were building exactly four shelving units
- **c.** exercise 2 of "Getting Started"

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 101–104.

You have now found the costs for most of the components of the shelves. Now, you will have to apply the formula for volume of a cylinder to answer the next question. There will be a little bit of thinking required to handle the missing part of the cylinder.

6. Turn to page 139 of the Project Book and answer exercises 3 and 4 of "Getting Started."

Compare your responses with the suggested answers in the Appendix, Activity 3, page 104.



Now that you know what the parts of the project cost, you will have to figure out how to put them all together and make a solid, usable shelving unit. Notice that there should be 12 nuts and 12 bolts in the supply list given for this set of shelves.

Turn to page 4 of Assignment Booklet 1A and answer question 7.

Customers will want a single, simple package to carry if they purchase a set of shelves. You will need to design a simple, small, cardboard container to hold the parts of this shelving unit.

Turn to pages 5 to 7 of Assignment Booklet 1A and answer questions 8, 9, and 10.

Looking Back

In this activity you explored the Fundamental Counting Principle. You used this principle to determine the number of outcomes in events and sample spaces and to calculate probabilities. You also completed a project that involved costing and packaging a home-assembly shelving system.

7. Turn to page 25 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 3, page 105.

Turn to pages 7 to 12 of Assignment Booklet 1A and answer questions 11 to 18.

ACTIVITY 4

Independent and Dependent Events

Suppose you flipped a coin and obtained seven heads in a row. Which do you think is more likely on the next toss: heads or tails? Contrary to what many people may think, if the coin is a fair coin and not weighted to favour one side or the other, heads and tails are equally likely on any given toss, regardless of what has occurred on previous tosses. The outcomes of consecutive tosses are example of **independent events**. Of course, you would expect an equal number of heads and tails over a large number of tosses, but it is a misconception that you can predict what will happen on a particular toss of the coin by studying patterns.

Now, suppose a bag contained an equal number of red marbles and white marbles, say 10 of each. If you reached in and selected 7 marbles, one at a time, without replacing them, and all of them are red, what colour marble is more likely to be drawn next, red or white? In this case, selecting a white marble is more probable because there are still 10 white marbles compared to only 3 red marbles. The outcomes of consecutive selections are examples of **dependent events**, because the outcome of one event affects the possible occurrence of another.

In this activity, you will examine independent and dependent events and how their probabilities are related.

Turn to page 28 of the textbook and read the introductory paragraph of Tutorial 1.4, "Independent and Dependent Events."

1. Complete exercises 1 to 4 of "Investigation 1: 'AND' Probabilities" on page 28 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 105–106.

A coin being tossed and a die being rolled is an example of independent events. The outcome of the coin toss, whether it is a head or a tail, does not affect the outcome of the roll of the die.

As you discovered in Investigation 1, for any two independent events, A and B, the probability that event A and event B both occur is given by the following formula:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example

Assuming that the birth of a boy and the birth of a girl are equally likely, what was the probability that the first two children born in 2002 were girls?

Solution

Method 1: Using the Formula

Let G_1 and G_2 represent the following events:

G_1: The first child was a girl.

G_2: The second child was a girl.

These events are independent, since the sex of the first child does not affect the sex of the second child.

Because the birth of a boy and the birth of a girl are equally likely,

$$P(G_1) = P(G_2) = \frac{1}{2} \text{ or } 0.5$$

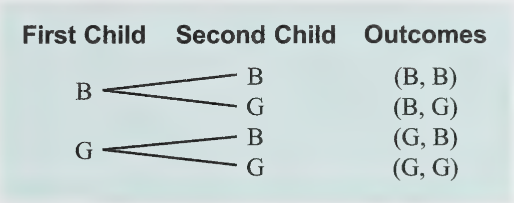
$$\begin{aligned}\therefore\ P(\text{two girls}) &= P(G_1) \times P(G_2)\\ &= \frac{1}{2} \times \frac{1}{2}\\ &= \frac{1}{4} \text{ or } 25\%\end{aligned}$$

or

$$\begin{aligned}P(\text{two girls}) &= P(G_1) \times P(G_2)\\ &= 0.5 \times 0.5\\ &= 0.25 \text{ or } 25\%\end{aligned}$$

Method 2: Using a Tree Diagram

Let B be the birth of a boy and G be the birth of a girl.

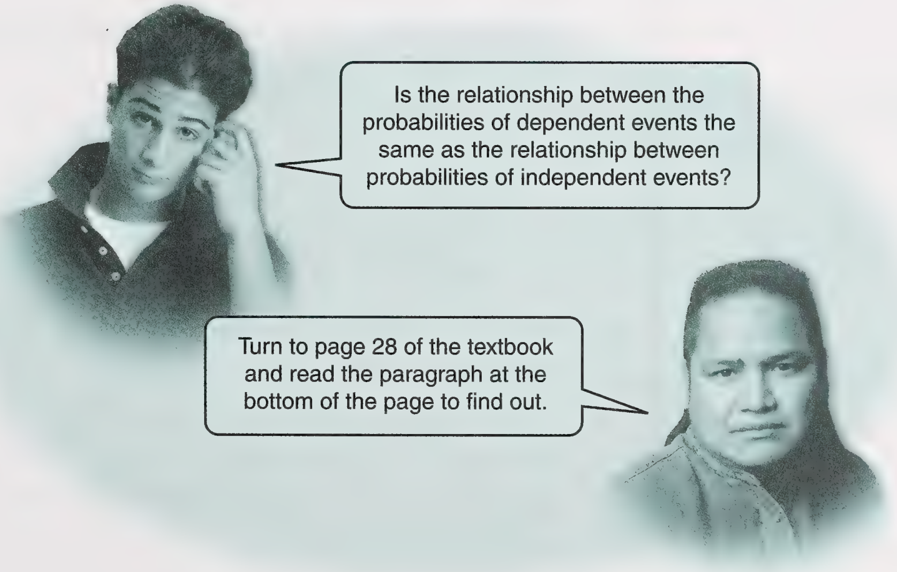


Because there are 4 equally likely outcomes and there is only 1 favourable outcome of the first two children being girls,

$$P(\text{two girls}) = \frac{1}{4} \text{ or } 25\%$$

Therefore, the probability that the first two children were girls is 25%.

Turn to pages 1 and 2 of Assignment Booklet 1B and answer question 1.


Is the relationship between the probabilities of dependent events the same as the relationship between probabilities of independent events?
Turn to page 28 of the textbook and read the paragraph at the bottom of the page to find out.

2. Complete exercises 1 to 4 of "Investigation 2: More 'AND' Probabilities" on page of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 106–107.

In Investigation 2 you discovered that for dependent events A and B, $P(\text{A and B}) \neq P(\text{A}) \times P(\text{B})$. Therefore, it is essential that you classify events as eithe dependent or independent.

Mathematicians use words very carefully when they are working with probability (and most other times, too). You have to be able to see and understand some very subtle differences in meaning to solve many types of problems.

Example

Four phrases are given.

A. at least 7
B. at most 7
C. no more than 7
D. no less than 7

Which of these phrases give you the same information?

Solution

Phrases A and D give the same information. Also, phrases B and C give the same information.

Example

Four phrases are given.

A. not less than 3
B. not more than 3
C. more than 2
D. less than 4

a. Which of these phrases give you the same information if you are dealing with the digits 0, 1, 2, 3, …, 8, 9?

b. If you were dealing with all real numbers between 0 and 9 inclusive, would this still be true?

Solution

a. Phrases A and C give the same information about the digits 0, 1, 2, …, 8, 9. Likewise, phrases B and D give the same information about the digits 0, 1, 2, …, 8, 9.

b. The same is not true if all real numbers between 0 and 9 are considered. For example, 2.5 is "more than 2" but is "less than 3." Similarly, 3.5 is "less than 4" but is "more than 3."

For more information on probability, view the segment *Probabilities and Medical Testing* on the Applied Mathematics 30 CD.

3. Turn to page 31 of the textbook and answer the following:

- **a.** exercises 1 and 3 of "Discussing the Ideas"
- **b.** exercise 1 of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 4, page 108.

Example

Two events, A and B, are independent. If $P(A \text{ and } B) = \frac{1}{10}$ and $P(B)$ is $\frac{1}{5}$, what is the probability of A, $P(A)$?

Solution

Events A and B are independent.

$$\begin{aligned} \therefore\ P(A \text{ and } B) &= P(A) \times P(B) \\ \frac{1}{10} &= P(A) \times \frac{1}{5} \\ &= \frac{1}{10} \div \frac{1}{5} \\ &= \frac{1}{2} \end{aligned}$$

The probability of A is $\frac{1}{2}$.

The formula $P(A \text{ and } B) = P(A) \times P(B)$ has three components: $P(A \text{ and } B)$, $P(A)$, and $P(B)$. If you know two of them, you can calculate the third.

In the following examples and exercises, you will determine probabilities that two or more events will occur. They are all examples of "and" probabilities.

Turn to pages 29 and 30 of the textbook and work through "Example 1: Determine the Probability of Tossing 4 Heads in a Row," "Example 2: Determine 'at least' Probabilities," and "Example 3: A Test for Independence."

4. Answer the following on pages 31 to 33 of the textbook.

- **a.** exercises 2 to 9 of "Exercises: Checking Your Skills"
- **b.** exercises 10 and 11 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 108–119.

Looking Back

In this activity you explored dependent and independent events and the relationship between their probabilities.

5. Turn to page 33 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 120.

ACTIVITY 5

Mutually Exclusive Events

Gloria is a mid-fielder on her field-hockey team. Her team has just made the league final. They will be playing a team from a neighbouring town for the league championship. During the regular season, the two teams met ten times and each won five games. Most believe the teams are still evenly matched, that either team has a 50% chance of winning the final and that the series will go seven games. However, there is a chance that one of the teams will win in fewer games. What is the probability that either Gloria's team or their opposition will sweep the series in four games?

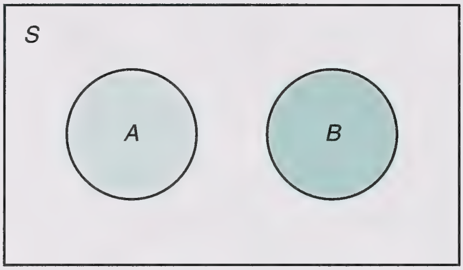
This scenario illustrates a probability question that involves two events linked by the word *or*. In this activity you will investigate "or" probabilities. As part of your investigation, you will need to determine if the events share any common outcomes.

Turn to page 34 of the textbook and read the introductory paragraph of Tutorial 1.5, "Mutually Exclusive Events."

1. Complete exercises 1 to 5 of "Investigation: 'OR' Probabilities" on page 34 of the textbook.

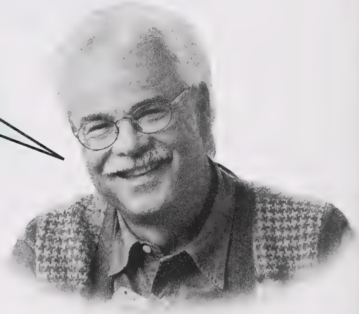
Compare your responses with the suggested answers in the Appendix, Activity 5, pages 120–122.

In the investigation, you were shown events that have common outcomes and others that have none. If event A and event B have no outcomes in common, they are called **mutually exclusive events**. Mutually exclusive events can be represented using a **Venn diagram**.



The rectangle in a Venn diagram contains all the outcomes that form the sample space of a particular experiment. The circles labelled A and B contain the outcomes of event A and event B. Since the two circles do not overlap, no outcomes are common to both events.

To determine the formula for the probability of Event *A* or Event *B* occurring, read on!



Example

Let A and B represent the following events when two dice are rolled:

A: "The sum of the dice is 7."
B: "The sum of the dice is 11."

a. How many outcomes are there in the sample space, S?
b. List the outcomes in event A, and determine the probability of event A.
c. List the outcomes in event B, and determine the probability of event B.
d. Are A and B mutually exclusive events? Why?
e. Determine the probability of throwing a 7 or 11 on the toss of two dice.
f. How are $P(A \text{ or } B)$, $P(A)$, and $P(B)$ related?

Solution

a. Each die can turn up 6 different ways.

Using the Fundamental Counting Principle, the two dice can turn up $6 \times 6 = 36$ different ways. Therefore, there are 36 outcomes in the sample space, *S*.

b. There are 6 outcomes in *A*: $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2)$, and $(6, 1)$.

$$\begin{aligned} \therefore P(A) &= \frac{6}{36} \\ &= \frac{1}{6} \end{aligned}$$

c. There are 2 outcomes in *B*: $(5, 6)$ and $(6, 5)$.

$$\begin{aligned} \therefore P(B) &= \frac{2}{36} \\ &= \frac{1}{18} \end{aligned}$$

d. Event *A* and event *B* are mutually exclusive because there are no common outcomes. Event *A* can occur or event *B* can occur, but both events cannot occur at the same time.

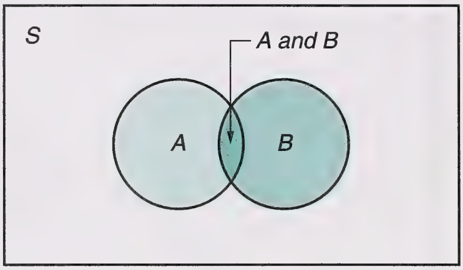
e. There are 6 ways of obtaining a sum of 7 and 2 ways of obtaining a sum of 11. Because all the outcomes are different, there are $6 + 2 = 8$ ways of obtaining a sum of 7 or 11.

$$\begin{aligned} \therefore P(A \text{ or } B) &= \frac{8}{36} \\ &= \frac{2}{9} \end{aligned}$$

f. Because $\frac{8}{36} = \frac{6}{36} + \frac{2}{36}$,

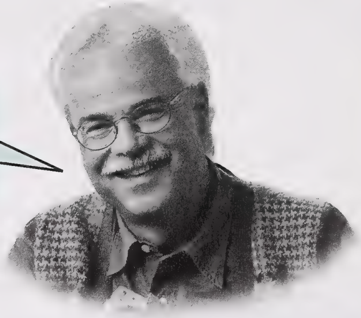
$$P(A \text{ or } B) = P(A) + P(B)$$

Not all events are mutually exclusive. Some events share outcomes. These events can also be pictured using a Venn diagram.



Events *A* and *B* in the diagram are not mutually exclusive. The circles representing these events overlap, indicating that some of the outcomes of event *A* are common to event *B*.

In the next example you will discover how to determine $P(A \text{ or } B)$ if *A* and *B* are not mutually exclusive.



Example

Two dice are tossed. Let *A* and *B* represent the following events:

A: "The sum of the dice is 10."
B: "Both dice are odd numbers."

a. Determine $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$.

b. How are the probabilities in exercise a. related?

Solution

a. Recall that there are $6 \times 6 = 36$ outcomes in the sample space when two dice are tossed.

There are 3 outcomes in *A*: $(4,6)$, $(5,5)$, and $(6,4)$.

$$\therefore P(A) = \frac{3}{36} = \frac{1}{12}$$

There are 9 outcomes in *B*: $(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),$ $(5,3)$, and $(5,5)$.

$$\begin{aligned}\therefore\ P(B) &= \frac{9}{36}\\ &= \frac{1}{4}\end{aligned}$$

Events *A* and *B* are not mutually exclusive; they can occur at the same time. Notice that $(5,5)$ is a common outcome.

$$\therefore\ P(A \text{ and } B) = \frac{1}{36}$$

There are 11 outcomes in *A* or *B*: $(4,6),(5,5),$ $(6,4),(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),$ $(5,1)$, and $(5,3)$.

Notice that $(5,5)$ was only counted once here.

$$\therefore\ P(A \text{ or } B) = \frac{11}{36}$$

b. Notice that $P(A \text{ or } B) \neq P(A) + P(B)$.

$$\frac{11}{36} \neq \frac{3}{36} + \frac{9}{36}$$

The sum of $P(A)$ and $P(B)$ is too large because the common outcome, $(5,5)$, has been counted twice: once in $P(A)$ and once in $P(B)$.

To compensate, an additional term is required.

$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B)\\ \frac{11}{36} &= \frac{3}{36} + \frac{9}{36} - \frac{1}{36}\end{aligned}$$

You have just been shown that you must determine $P(A \text{ and } B)$ before you can find $P(A \text{ or } B)$ if events *A* and *B* are not mutually exclusive. So, before you can answer an "or" probability question, you must answer an "and" probability question if the events are not mutually exclusive.

First, you need to practise classifying events as mutually exclusive or not mutually exclusive.

2. Turn to pages 37 and 38 of the textbook and answer exercises 1 and 2 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 5, page 122.

Example

If events A and B are mutally exclusive, what is $P(B)$ if $P(A \text{ or } B) = \frac{7}{8}$ and $P(A) = \frac{3}{4}$?

Solution

For mutually exclusive events A and B,

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ \frac{7}{8} &= \frac{3}{4} + P(B) \\ P(B) &= \frac{7}{8} - \frac{3}{4} \\ &= \frac{7}{8} - \frac{6}{8} \\ &= \frac{1}{8} \end{aligned}$$

The probability of event B is $\frac{1}{8}$.

The equation $P(A \text{ or } B) = P(A) + P(B)$ has three components: $P(A \text{ or } B)$, $P(A)$, and $P(B)$. If you know the value of two of them, you can calculate the third.

Turn to pages 34 and 35 of the textbook and read the text following the investigation. Then work through "Example 1: Using a Computer" and "Example 2: Combining 'and' and 'or' Probabilities" on pages 36 and 37.

3. Answer exercises 3 to 8 of "Exercises: Checking Your Skills" on pages 38 and 39 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 123–133.

Turn to pages 2 and 3 of Assignment Booklet 1B and answer questions 2 and 3.

Looking Back

In this activity you explored mutually exclusive events, events that are not mutually exclusive, and the relationship between their probabilities when solving "or" probability problems.

4. Turn to page 39 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 133.

Module Review

This module dealt with Chapter 1: Probability in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 42 of the textbook and notice the skills and concepts that were developed in this module. Also, read the important results and formulas that you discovered.

1. Answer exercises 1.a. to 1.f. of Part A of "What Should I Be Able to Do?" on page 43 of the textbook.

2. Answer exercises 2 to 8 of Part B of "What Should I Be Able to Do?" on pages 44 and 45 of the textbook.

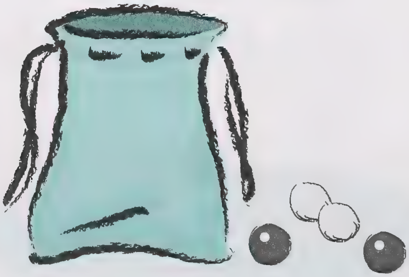
Compare your responses with the suggested answers in the Appendix, Module Review, pages 134–141.

Turn to pages 3 to 12 of Assignment Booklet 1B and complete the Module Review Assignment.

If you had difficulties understanding the skills and concepts in Module 1: Probability, it recommended that you contact your teacher for some extra help activities. If you have a clear understanding of the skills and concepts in this module, you may wish to do the following enrichment activity. You may decide to do both.

Enrichment

In probability theory, if the probability of one event is altered by the occurrence or non-occurrence of another event, the probability is described as **conditional probabilit** A simple example of conditional probability involves the selection of coloured marbles from a bag.



Example

A bag contains 2 red marbles and 2 white marbles. Two marbles are selected, one at a time, without replacement. What is the probability of drawing a red marble on the second draw, if

a. a red marble is selected on the first draw
b. a white marble is selected on the first draw

Solution

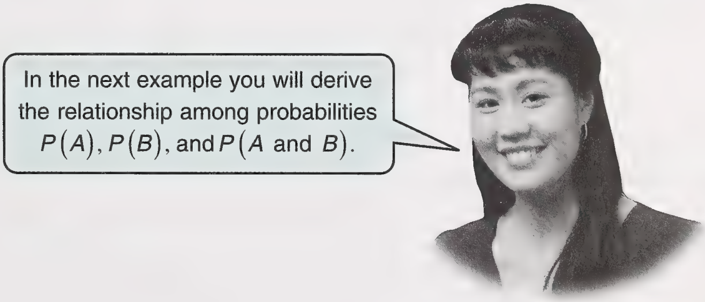
a. If a red marble is selected on the first draw, there is only 1 red marble left in the bag. Because the first marble is not replaced, there are only 3 marbles in the bag. Therefore, the probability of selecting a red marble on the second draw is $\frac{1}{3}$.

b. If a white marble is selected on the first draw, there are still 2 red marbles left in the bag. Therefore, the probability of selecting a red marble on the second draw is $\frac{2}{3}$.

Recall from Activity 4 that if the probability of one event is conditional on the occurrence of a second event, the events are classified as dependent events. The calculation of the probabilities of dependent events is an application of conditional probability.

In the preceding example, suppose event *A* was selecting a red marble on the first draw and event *B* was selecting a red marble on the second draw. As you determined, the probability of event *B* was $\frac{1}{3}$ if event *A* occurred.

The symbol for this probability is $P(B|A)$ and is read "the probability of event *B* given event *A* has occurred," or simply, "the probability of *B* given *A*." Of course, if both events occur, you will end up with two red marbles.


In the next example you will derive the relationship among probabilities $P(A)$, $P(B)$, and $P(A \text{ and } B)$.

Example

A bag contains 2 red marbles and 2 white marbles. Two marbles are selected, one at a time, without replacement.

a. What is the probability of drawing two red marbles?

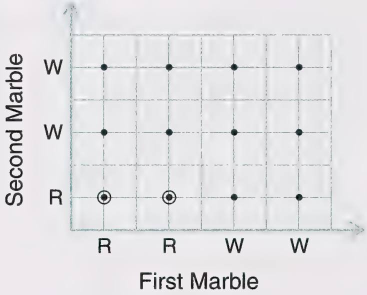
b. Let event A represent drawing a red marble on the first draw. Let event $B|A$ represent drawing a red marble on the second draw given a red marble was drawn on the first draw. Let event A and B represent drawing a red marble on both draws.

What is the relationship among probabilities $P(A)$, $P(B|A)$, and $P(A \text{ and } B)$?

Solution

a. There are 4 marbles for the first draw. Because you are interested in obtaining 2 red marbles and because the first marble drawn is not replaced, there will be 3 marbles—1 red, *R*, and 2 white, *W*—available for the second draw. Using the Fundamental Counting Principle, there are $4 \times 3 = 12$ outcomes in the sample space.

Use a grid to represent the outcomes.



There are 2 outcomes of the form, (R, R); that is, a red marble on the first draw and a red marble on the second draw.

$$\therefore P(2\text{R}) = \frac{2}{12}$$

$$= \frac{1}{6}$$

b. The probability of selecting a red marble on the first draw, $P(A)$, is $\frac{2}{4}$ or $\frac{1}{2}$. The probability of selecting a red marble on the second draw, given that a red marble was selected on the first draw, $P(B|A)$, is $\frac{1}{3}$.

The probability of drawing two red marbles is $P(A \text{ and } B) = \frac{1}{6}$. The relationship among these probabilities is as follows.

$$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$$
$$P(A \text{ and } B) = P(A) \times P(B|A)$$

In the preceding example, you discovered the relationship between the probabilities of two dependent events, *A* and *B*.

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

This formula is referred to as **Baye's Formula**. In the next example you will apply Baye's Formula.

Example

Two cards are drawn from a standard deck, one at a time, without replacement.

a. What is the probability of drawing two aces?
b. What is the probability of drawing an ace and a king?

Solution

a. Let $P(\mathrm{A}_1)$ be the probability of drawing an ace on the first draw. Because there are 4 aces among the 52 cards, $P(\mathrm{A}_1) = \frac{4}{52}$ or $\frac{1}{13}$.

Let $P(\mathrm{A}_2|\mathrm{A}_1)$ be the probability of drawing an ace on the second draw given that an ace was selected on the first draw. For the second draw, there are only 3 aces among the 51 cards remaining in the deck.

$$\therefore P(\mathrm{A}_2|\mathrm{A}_1) = \frac{3}{51} \text{ or } \frac{1}{17}$$

Use Baye's Formula to determine the probability of drawing two aces.

$$\begin{aligned} P(\text{A}_1 \text{ and } \text{A}_1) &= P(\text{A}_1) \times P(\text{A}_2 | \text{A}_1) \\ &= \frac{1}{13} \times \frac{1}{17} \\ &= \frac{1}{221} \text{ or about } 0.5\% \end{aligned}$$

The probability of drawing two aces is $\frac{1}{221}$, or about 0.5%.

b. Let $P(\text{A}_1)$ be the probability of drawing an ace on the first draw. Because there are 4 aces among the 52 cards, $P(\text{A}_1) = \frac{4}{52}$ or $\frac{1}{13}$.

Let $P(\text{K}_2 | \text{A}_1)$ be the probability of drawing a king on the second draw, given that an ace was selected on the first draw. For the second draw, there are 4 kings among the 51 cards remainng.

$$\therefore P(\text{K}_2 | \text{A}_1) = \frac{4}{51}$$

Use Baye's Formula for determining the probability of drawing an ace followed by a king.

$$\begin{aligned} P(\text{A}_1 \text{ and } \text{K}_2) &= P(\text{A}_1) \times P(\text{K}_2 | \text{A}_1) \\ &= \frac{1}{13} \times \frac{4}{51} \\ &= \frac{4}{663} \end{aligned}$$

However, it is equally likely that you could draw a king first followed by an ace.

$$\begin{aligned} P(\text{K}_1 \text{ and } \text{A}_2) &= P(\text{K}_1) \times P(\text{A}_2 | \text{K}_1) \\ &= \frac{1}{13} \times \frac{4}{51} \\ &= \frac{4}{663} \end{aligned}$$

$$\begin{aligned}\therefore\ P(\text{ace and king}) &= P(\text{A}_1 \text{ and K}_2 \text{ or K}_1 \text{ and A}_2) \\ &= P(\text{A}_1 \text{ and K}_2) + P(\text{K}_1 \text{ and A}_2) \\ &= \frac{4}{663} + \frac{4}{663} \\ &= \frac{8}{663} \text{ or about } 1.2\%\end{aligned}$$

The probability of drawing an ace and a king is $\frac{8}{663}$, or about 1.2%.

In each the following questions, use Baye's Formula to determine the required conditional probability.

1. Three cards are drawn without replacement from a deck of cards. What is the probability of drawing three face cards?

2. If three cards are drawn without replacement from a deck of cards, what is the probability of drawing one king and two queens?

3. A bag contains three red marbles and two white marbles. Two marbles are selected without replacement. What is the probability of drawing one red marble and one white marble?

Compare your responses with the suggested answer in the Appendix, Module Review: Enrichment, pages 142–143.

MODULE PROJECT

Voice-Mail Security

Completing the Project

By now you should have completed the initial research for the Module 1 project, Voice-Mail Security. You should have a good understanding of password systems for accessing voice-mail messages.

For this project, you will be asked to complete two parts. First, you will determine the number of possible passwords for each of two proposals for the voice-mail system of a small telephone company. Then you will make recommendations and suggest improvements to these proposals. You will base your recommendations, in part, on the probability that voice-mail users are likely to forget their passwords.

Turn to pages 26 and 27 of the textbook and read "How Many Passwords Are Possible?" Answer exercises 1 to 4 of Proposal A and exercises 1 to 5 of Proposal B. Keep your responses to these exercises in the project section of your mathematics binder. You will use them later to help you complete the questions in the Module Project section of Assignment Booklet 1B.

Before you continue your module project, look at the sample project in Part C of "What Should I Be Able to Do?" on pages 45 to 48 of the textbook.

1. Turn to page 46 of the textbook and answer the first bulleted question of exercise 9 of Part C of “What Should I Be Able to Do?”

   Note: Keep a copy of your response to the preceding exercise in the project section of your mathematics binder. You may need this information to determine the number of possible passwords for Proposal A and Proposal B in Assignment Booklet 1B.

2. Turn to pages 47 and 48 of the textbook and answer exercise 10 of Part C of “What Should I Be Able to Do?”

Compare your response with the suggested answer in the Appendix, Module Project, pages 144–148.

Module Project

Now that you have more insight into the module project, revise your answers. Then complete the Module 1 project, Voice-Mail Security. You may use your responses from the textbook exercises on pages 2, 26 and 27, and 46 and 47 to help you complete the project. (Your responses should be in the project section of your mathematics binder.)

Turn to pages 13 to 18 of Assignment Booklet 1B and complete the module project.

MODULE SUMMARY

In this module you explored experimental and theoretical probability. You determined theoretical probabilities of particular events by analysing the number of outcomes favourable to these events as a proportion of the total number of possible outcomes. You investigated techniques for determining numbers of outcomes, including tree diagrams, tables, grids, and the Fundamental Counting Principle.

As part of your investigation, you looked at related events. You classified these events as complementary, mutually exclusive, not mutually exclusive, dependent, or independent. You used these classifications to help you decide how to approach probability problems, particularly those that involved key words like *not*, *at least*, *at most*, *and*, and *or*.

The study of probability has a variety of applications that include games of chance, insurance rates, surveys and polls, medicine, and quality control to name a few. One application you explored in detail was the design of password security systems for telephone voice mail. Security and privacy should be a matter of applied mathematics; it should never be left to chance!

APPENDIX

Glossary
Calculator Functions
Suggested Answers
Coin Toss
Image Credits

Glossary

complement of an event: all the outcomes in a sample space that do not belong to the original event

dependent events: two or more events for which the occurrence of one affects the probability of the occurrence of the others

event: an outcome or a set of outcomes of an experiment

experimental probability: probability determined by observing the relative frequency of a given event

Fundamental Counting Principle: a technique for determining the number of ways two or more tasks can be performed by multiplying the number of ways each separate task can be performed

independent events: two or more events for which the occurrence of one does not affect the occurrence of the others

mutually exclusive events: events that cannot occur at the same time

outcome: the result of an experiment

probability: the ratio of the number of favourable outcomes to the total number of possible outcomes

sample space: the set of all possible outcomes of an experiment

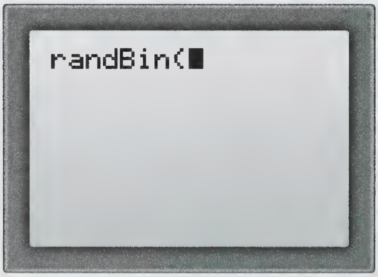
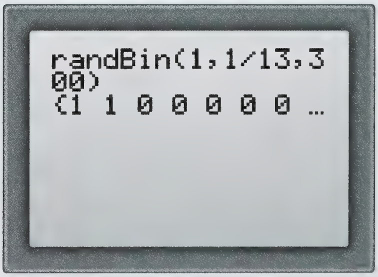
tree diagram: a branching diagram illustrating all of the outcomes of an event or related events

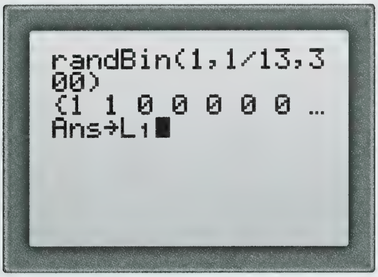
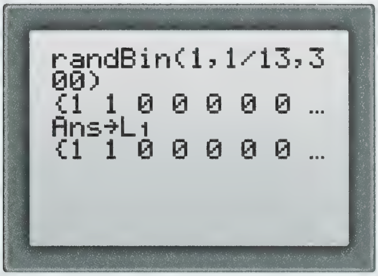
theoretical probability: probability determined by a theoretical approach to the relative frequency of an event

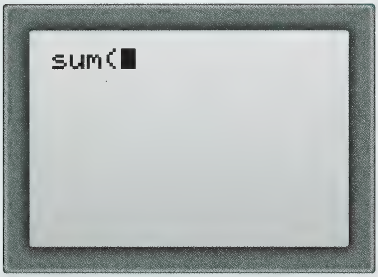
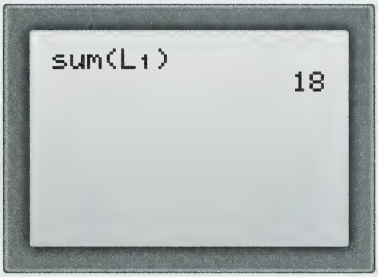
Venn diagram: a diagram using circles inside a rectangle to model events

Calculator Functions

Function	Keystrokes	Example

randBin	select the PRB menu
	MATH ◀ 7 (7:randBin()
	
	`randBin(`
		
		`randBin(1,1/13,300)`
		`{1 1 0 0 0 0 0 …`

Storing data in a list	STO➔ 2nd [L1]
	
	`randBin(1,1/13,3`
	`00)`
	`{1 1 0 0 0 0 0 …`
	`Ans→L1`
		
		`randBin(1,1/13,3`
		`00)`
		`{1 1 0 0 0 0 0 …`
		`Ans→L1`
		`{1 1 0 0 0 0 0 …`

sum	Select the MATH menu.
	STAT ◀ 5 (5:sum()
	
	`sum(`
		
		`sum(L1)`
		`18`

To review how to obtain other calculator functions, refer to the Calculator folder on the Applied Mathematics 30 CD.

Suggested Answers

Activity 1: Experimental and Theoretical Probability

1. **Textbook exercises 1 to 3 of "Investigation: Finding Experimental and Theoretical Probabilities," pp. 4 and 5**

    1. Answers will vary. A sample answer is given. The results of the coin tosses are as follows:

HTT	HHH	THT	TTT	HTH	HTH	TTT	THT	HHT	HHH
HHH	HHH	HHT	THT	THH	HTT	TTT	HHT	HTT	HHH
THH	HHH	TTH	HTT	HHT	HTH	HTH	TTH	TTT	TTH

There were 10 out of a 30 tosses that had exactly two heads.

$$\begin{aligned}\therefore P(\text{exactly 2H}) &= \frac{10}{30}\\ &= \frac{1}{3} \text{ or } 0.3\overline{3}\end{aligned}$$

Activity 1 (continued)

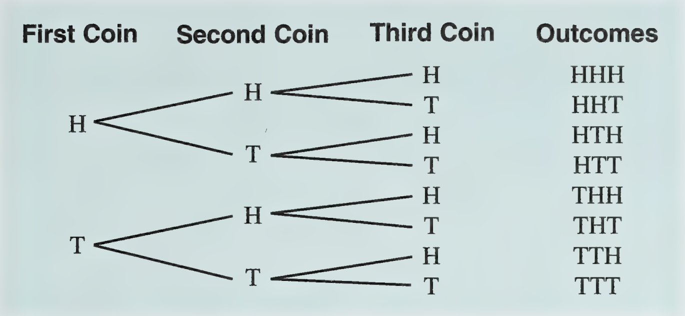
2. Answers will vary. A sample answer is given based on the Coin Toss data given on pages 148 and 149 of the Appendix.

Group	Number of Results with Exactly 2 Heads	Total Number of Results
1	9	30
2	7	30
3	9	30
4	13	30
5	9	30
6	13	30
7	10	30
8	11	30
9	14	30
10	16	30
11	14	30
	13	30
13	13	30
14	19	30
15	10	30
16	7	30
17	8	30
18	13	30
19	9	30
20	12	30
21	12	30
22	14	30
23	11	30
24	14	30
25	6	30
Totals	286	750

$$\therefore P(\text{exactly } 2\text{H}) = \frac{286}{750} \text{ or about } 0.38$$

This is close to the probability determined in exercise 1.

3.



There are 8 possible outcomes, 3 of which have exactly two heads. This gives a theoretical probability of $\frac{3}{8}$, or 0.375. This is close to both probabilities found in exercises 1 and 2.

2. **a.** This is a theoretical probability. A fair die has six faces, one of which is a 1. Therefore, the theoretical probability is $\frac{1}{6}$.

b. This is an example of experimental probability. The probability that a newly licensed driver will make a claim is based on statistics collected by the insurance company. The probability is based on experience rather than theory.

3. **a.** **Textbook exercises 2 and 3 of "Discussing the Ideas," p. 9**

2. Answers will vary. A sample answer is given.

A test for a particular disease has been developed. When the test is administered, a person can test positive or negative. Suppose that 9 out of 10 people who test positive have the disease. Therefore, the probability that a person who tests positive has the disease is $\frac{9}{10}$ or 90%. This is an example in which experimental probability would have to be used.

3. Your friend's comment was based on only 10 trials. This is far too few to challenge the accuracy of the theoretical probability.

b. **Textbook exercises 1 to 10 of "Exercises: Checking Your Skills," pp. 10 to 12**

1. **a.**

$$\begin{aligned} P(\text{head}) &= \frac{\text{Number of heads}}{\text{Number of tosses}} \\ &= \frac{40}{60} \\ &= \frac{2}{3} \text{ or about } 66.7\% \end{aligned}$$

The experimental probability is $\frac{2}{3}$, or about 66.7%.

b. Because a fair coin can turn up heads or tails on each toss, the theoretical probability is

$$P(\text{head}) = \frac{1}{2} \text{ or } 50\%$$

2. **a.** $P(\text{winning}) = \frac{1}{250} \text{ or } 0.4\%$

b.
$$\begin{aligned} P(\text{winning}) &= \frac{5}{250} \\ &= \frac{1}{50} \text{ or } 2\% \end{aligned}$$

c.
$$\begin{aligned} P(\text{winning}) &= \frac{10}{250} \\ &= \frac{1}{25} \text{ or } 4\% \end{aligned}$$

3. **a.** There are four 7s in the deck.

$$\begin{aligned} \therefore P(7) &= \frac{4}{52} \\ &= \frac{1}{13} \text{ or about } 7.7\% \end{aligned}$$

b. There are 13 diamonds in the deck.

$$\begin{aligned} \therefore P(\text{diamond}) &= \frac{13}{52} \\ &= \frac{1}{4} \text{ or } 25\% \end{aligned}$$

c. There are 26 red cards in a deck.

$$\begin{aligned} \therefore P(\text{red card}) &= \frac{26}{52} \\ &= \frac{1}{2} \text{ or } 50\% \end{aligned}$$

d. There are two black 10s in a deck.

$$\begin{aligned} \therefore P(\text{black } 10) &= \frac{2}{52} \\ &= \frac{1}{26} \text{ or about } 3.8\% \end{aligned}$$

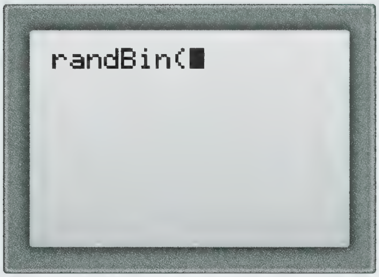
4. Answers will vary. Sample simulations are given. **Remember:** The outcomes of the simulations given will differ slightly from your outcomes because the graphing calculator chooses the outcomes randomly.

a. The theoretical probability of drawing a 7 from a standard deck of cards is $\frac{1}{13}$.

Step 1: Select the Random Binary Number feature.

Select the PRB menu.

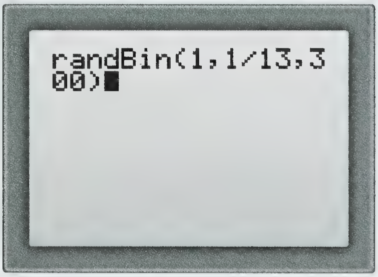
MATH ◀ 7 (7:randBin()



```
randBin(█
```

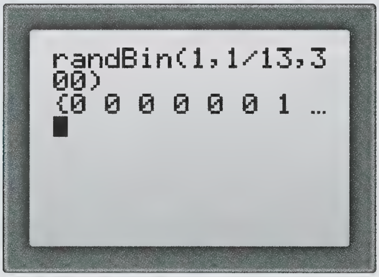
Step 2: Enter the information for the simulation.

1 , 1 ÷ 1 3 , 3 0 0)



```
randBin(1,1/13,3
00)█
```

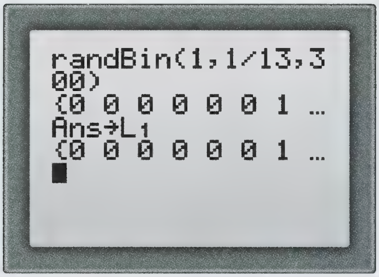
Step 3: Press ENTER to obtain the outcomes of the simulation.



```
randBin(1,1/13,3
00)
{0 0 0 0 0 0 1 …
```

Step 4: Store the outcomes as a list.

2nd [ANS] STO➔ 2nd [L1] ENTER

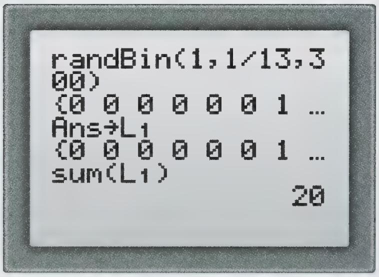


```
randBin(1,1/13,3
00)
{0 0 0 0 0 0 1 …
Ans→L1
{0 0 0 0 0 0 1 …
```

Step 5: Determine the sum of the list.

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L1]) ENTER



```
randBin(1,1/13,3
00)
{0 0 0 0 0 0 1 …
Ans→L1
{0 0 0 0 0 0 1 …
sum(L1)
                 20
```

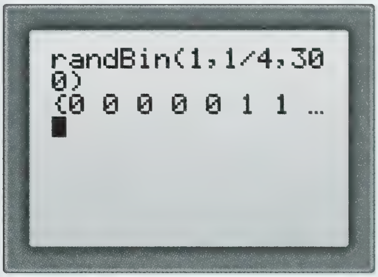
In this simulation, a 1 occurred 20 out of 300 times. Therefore, the experimental probability of drawing a 7 is $\frac{20}{300} = \frac{1}{15}$, or about 6.7%.

b. The theoretical probability of drawing a diamond from a standard deck of cards is $\frac{1}{4}$.

Select the Random Binary Number feature; enter the information; and obtain the outcomes for the simulation.

Select the PRB menu

MATH ◀ 7 (7:randBin() 1 , 1 ÷ 4 , 3 0 0) ENTER



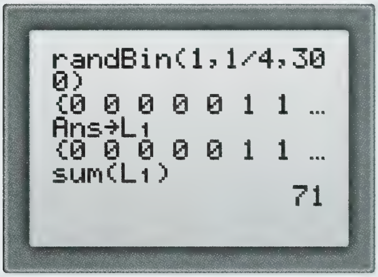
```
randBin(1,1/4,30
0)
{0 0 0 0 0 1 1 …
█
```

Store the outcomes as a list; then determine the sum of the list.

2nd [ANS] STO➔ 2nd [L1] ENTER

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L1]) ENTER



```
randBin(1,1/4,30
0)
{0 0 0 0 0 1 1 …
Ans→L1
{0 0 0 0 0 1 1 …
sum(L1)
              71
```

In this simulation, a 1 occurred 71 out of 300 times.

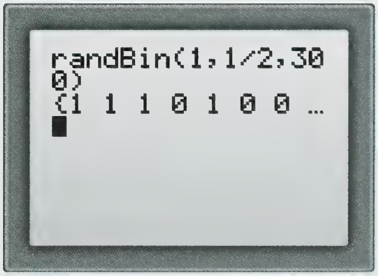
Therefore, the experimental probability of drawing a diamond is $\frac{71}{300}$, or about 23.7%.

c. The theoretical probability of drawing a red card from a standard deck of cards is $\frac{1}{2}$.

Select the Random Binary Number feature; enter the information; and obtain the outcomes for the simulation.

Select the PRB menu.

MATH ◀ 7 (7:randBin() 1 , 1 ÷ 2 , 3 0 0) ENTER



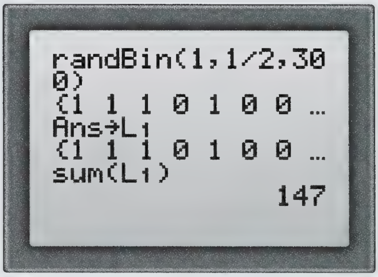
```
randBin(1,1/2,30
0)
{1 1 1 0 1 0 0 …
█
```

Store the outcomes as a list; then determine the sum of the list.

2nd [ANS] STO➧ 2nd [L1] ENTER

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L1]
) ENTER



```
randBin(1,1/2,30
0)
{1 1 1 0 1 0 0 …
Ans→L1
{1 1 1 0 1 0 0 …
sum(L1)
             147
```

In this simulation, a 1 occurred 147 out of 300 times. Therefore, the experimental probability of drawing a red card is $\frac{147}{300} = \frac{49}{100}$, or 49%.

d. The theoretical probability of drawing a black 10 from a standard deck of cards is $\frac{1}{26}$.

Perform a simulation on your graphing calculator, and determine the experimental probability.

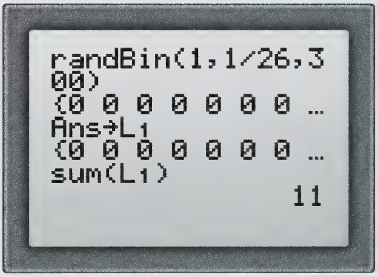
Select the PRB menu.

MATH ◀ 7 (7:randBin() 1 , 1
÷ 2 6 , 3 0 0) ENTER

2nd [ANS] STO➧ 2nd [L1] ENTER

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L1]
) ENTER



```
randBin(1,1/26,3
00)
{0 0 0 0 0 0 0 …
Ans→L1
{0 0 0 0 0 0 0 …
sum(L1)
              11
```

In this simulation, a 1 occurred 11 out of 300 times. Therefore, the experimental probability of drawing a black 10 is $\frac{11}{300}$, or about 3.7%.

5. The traffic signal completes one cycle in $30+5+45=80$ s.

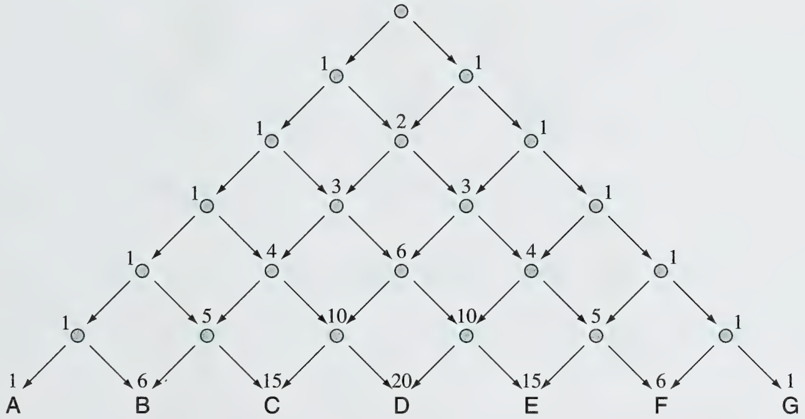
The signal is red for 30 s.

$$\begin{aligned}\therefore\ P(\text{red}) &= \frac{30}{80}\\ &= \frac{3}{8} \text{ or } 37.5\%\end{aligned}$$

6. Among the numbers 1 to 36, there are 18 even numbers and 18 odd numbers. There are 38 numbers in total on the roulette wheel.

$$\begin{aligned}\therefore\ P(\text{even}) &= \frac{18}{38}\\ &= \frac{9}{19} \text{ or about } 47.4\%\end{aligned}$$

7. a.

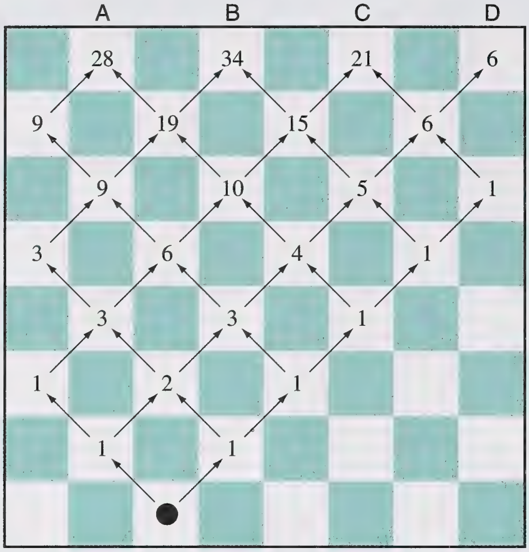


There is 1 pathway leading to A, 6 pathways leading to B, 15 pathways leading to C, 20 pathways leading to D, 15 pathways leading to E, 6 pathways leading to F, and 1 pathway leading to G.

b. There are 64 pathways altogether.

$$\begin{aligned}\therefore\ P(\text{exit at D}) &= \frac{20}{64}\\ &= \frac{5}{16} \text{ or } 31.25\%\end{aligned}$$

8. **a.** Determine the number of paths as you move the game piece diagonally upward.



There are 28 pathways to A, 34 pathways to B, 21 pathways to C, and 6 pathways to D.

b. **i.** $P(A)=\frac{28}{89}$ or about 31.5%

ii. $P(C)=\frac{21}{89}$ or about 23.6%

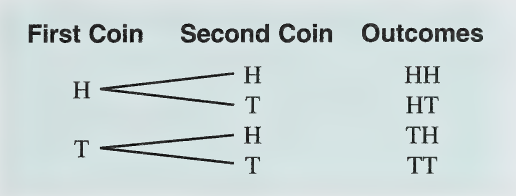
9. **a.** Answers will vary. The results of a sample experiment of tossing 2 coins 20 times are given.

Outcome	Tally	Total
2 Heads	||||	4
1 Head, 1 Tail	𝍸 𝍸 |	11
2 Tails	𝍸	5

In this experiment, two heads occurred 4 times. Therefore, the experimental probability is

$$P(2\text{ heads})=\frac{4}{20}$$
$$=\frac{1}{5}\text{ or }20\%$$

b. Draw a tree diagram.



There are four outcomes altogether, one of which consists of two heads. Therefore, the theoretical probability is

$$P(2 \text{ heads}) = \frac{1}{4} \text{ or } 25\%$$

c. The results are close. They differ by 5%. The difference should decrease with more trials.

10. a. i. To win, the card dealt must be a 3, 4, or 5. Because there are 4 of each of these cards in the deck, there are 12 winning cards out of the 50 remaining.

$$\begin{aligned} \therefore P(\text{winning}) &= \frac{12}{50} \\ &= \frac{6}{25} \text{ or } 24\% \end{aligned}$$

ii. The winning card must be a 6, 7, 8, 9, 10, or jack. Therefore, there are $4 \times 6 = 24$ winning cards.

$$\begin{aligned} \therefore P(\text{winning}) &= \frac{24}{50} \\ &= \frac{12}{25} \text{ or } 48\% \end{aligned}$$

iii. There are no cards between a 7 and an 8.

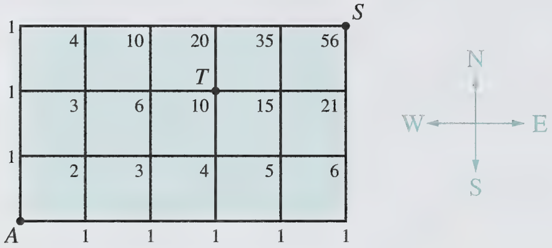
$$\therefore P(\text{winning}) = \frac{0}{50} \text{ or } 0\%$$

iv. The winning card is any one of four queens.

$$\begin{aligned} \therefore P(\text{winning}) &= \frac{4}{50} \\ &= \frac{2}{25} \text{ or } 8\% \end{aligned}$$

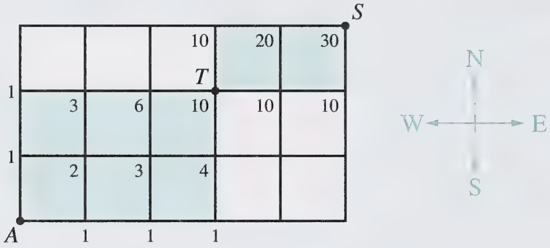
b. The answers to question 10.a. assume that the first two cards are not replaced before the third card is dealt.

4. Determine the total number of routes from Antonella's home to her school.



There are 56 different routes to the school from Antonella's home.

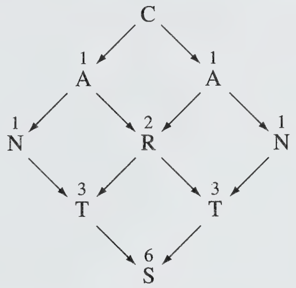
Now, determine the number of routes to the school from Antonella's home that will take her past the town hall, *T*.



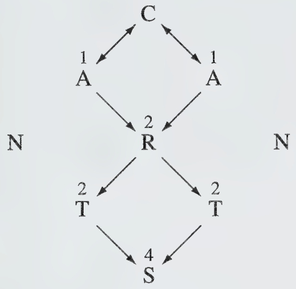
There are 30 different routes to the school from Antonella's home that will take her past the town hall.

$$\begin{aligned}\therefore\ P(\text{route past } T) &= \frac{30}{56}\\ &= \frac{15}{28}\ \text{or about } 53.6\%\end{aligned}$$

5. Determine the number of pathways from C to S.



There are 6 ways of reaching the letter S. For the path to spell CARTS, it must pass through the letter R. Determine the number of pathways through R.



There are 4 pathways that spell CARTS.

$$\begin{aligned}\therefore\ P(\text{CARTS}) &= \frac{4}{6} \\ &= \frac{2}{3}\end{aligned}$$

6. Textbook exercise "Communicating the Ideas," p. 12

Answers will vary. A sample response is given.

The theoretical probability of a head on the toss of a coin is $\frac{1}{2}$. This means heads and tails are equally likely to occur. However, this does not mean the coin will land heads once for every two flips of a coin. If the coin lands heads on the first toss, there is still a 50% probability of landing heads on the second toss, third toss, and so on. The coin could land heads on every toss you try. You cannot predict what the outcome will be of any toss!

Activity 2: Generating and Using Sample Spaces

1. **Textbook exercises 1 to 8 of "Investigation: Rolling Two Dice," pp. 13 and 14**

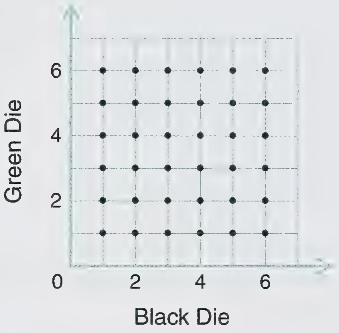
1.

Black Die

Green Die	1	2	3	4	5	6
1	$(1,1)$	$(2,1)$	$(3,1)$	$(4,1)$	$(5,1)$	$(6,1)$
2	$(1,2)$	$(2,2)$	$(3,2)$	$(4,2)$	$(5,2)$	$(6,2)$
3	$(1,3)$	$(2,3)$	$(3,3)$	$(4,3)$	$(5,3)$	$(6,3)$
4	$(1,4)$	$(2,4)$	$(3,4)$	$(4,4)$	$(5,4)$	$(6,4)$
5	$(1,5)$	$(2,5)$	$(3,5)$	$(4,5)$	$(5,5)$	$(6,5)$
6	$(1,6)$	$(2,6)$	$(3,6)$	$(4,6)$	$(5,6)$	$(6,6)$

2. There are 36 possible outcomes for this experiment.

3.



4. **a.** Event B: Roll the same number on each die (doubles).

Black Die

Green Die

	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	$(1,1)$	$(2,1)$	$(3,1)$	$(4,1)$	$(5,1)$	$(6,1)$
[illegible]	$(1,2)$	$(2,2)$	$(3,2)$	$(4,2)$	$(5,2)$	$(6,2)$
[illegible]	$(1,3)$	$(2,3)$	$(3,3)$	$(4,3)$	$(5,3)$	$(6,3)$
[illegible]	$(1,4)$	$(2,4)$	$(3,4)$	$(4,4)$	$(5,4)$	$(6,4)$
[illegible]	$(1,5)$	$(2,5)$	$(3,5)$	$(4,5)$	$(5,5)$	$(6,5)$
[illegible]	$(1,6)$	$(2,6)$	$(3,6)$	$(4,6)$	$(5,6)$	$(6,6)$

b. There are 6 favourable outcomes for event B.

$$\begin{aligned}\therefore\ P(\text{B}) &= \frac{6}{36}\\ &= \frac{1}{6}\end{aligned}$$

c. There are 30 outcomes not favourable to event B.

$$\begin{aligned}\therefore\ P(\text{not B}) &= \frac{30}{36}\\ &= \frac{5}{6}\end{aligned}$$

5. **a.** Event C: The sum of the two dice is 5.

Black Die

Green Die

	1	2	3	4	5	6
1	$(1, 1)$	$(2, 1)$	$(3, 1)$	$\boxed{(4, 1)}$	$(5, 1)$	$(6, 1)$
2	$(1, 2)$	$(2, 2)$	$\boxed{(3, 2)}$	$(4, 2)$	$(5, 2)$	$(6, 2)$
3	$(1, 3)$	$\boxed{(2, 3)}$	$(3, 3)$	$(4, 3)$	$(5, 3)$	$(6, 3)$
4	$\boxed{(1, 4)}$	$(2, 4)$	$(3, 4)$	$(4, 4)$	$(5, 4)$	$(6, 4)$
5	$(1, 5)$	$(2, 5)$	$(3, 5)$	$(4, 5)$	$(5, 5)$	$(6, 5)$
6	$(1, 6)$	$(2, 6)$	$(3, 6)$	$(4, 6)$	$(5, 6)$	$(6, 6)$

b. There are 4 favourable outcomes to event C.

$$\begin{aligned} \therefore P(\mathrm{C}) &= \frac{4}{36} \\ &= \frac{1}{9} \end{aligned}$$

c. There are 32 outcomes not favourable to event C.

$$\begin{aligned} \therefore P(\text{not C}) &= \frac{32}{36} \\ &= \frac{8}{9} \end{aligned}$$

6. **a.** Event D: The sum of the two dice is greater than 9.

Black Die

Green Die	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

b. There are 6 favourable outcomes to event D.

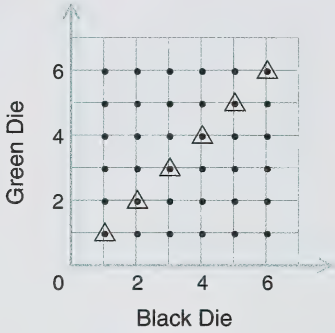
$$\therefore P(\text{D}) = \frac{6}{36}$$
$$= \frac{1}{6}$$

c. There are 30 outcomes not favourable to event D.

$$\therefore P(\text{not D}) = \frac{30}{36}$$
$$= \frac{5}{6}$$

7. Repeat exercises 4 to 6 using a grid.

4. **a.** Event B: Roll the same number on each die (doubles).



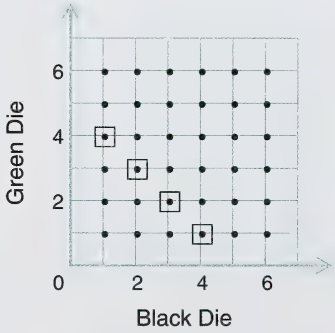
b. There are 6 points on the grid that represent doubles.

$$\begin{aligned}\therefore\ P(\mathrm{B}) &= \frac{6}{36}\\ &= \frac{1}{6}\end{aligned}$$

c. There are 30 points that do not represent doubles.

$$\begin{aligned}\therefore\ P(\text{not B}) &= \frac{30}{36}\\ &= \frac{5}{6}\end{aligned}$$

5. **a.** Event C: The sum of the two dice is 5.



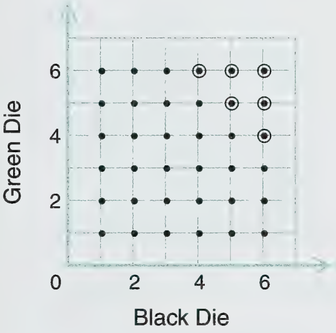
b. There are 4 points on the grid that represent a sum of 5.

$$\therefore P(\text{C}) = \frac{4}{36}$$
$$= \frac{1}{9}$$

c. There are 32 points on the grid that do not represent a sum of 5.

$$\therefore P(\text{not C}) = \frac{32}{36}$$
$$= \frac{8}{9}$$

6. **a.** Event D: The sum of the two dice is greater than 9.



b. There are 6 points on the grid that represent a sum greater than 9.

$$\therefore P(\text{D}) = \frac{6}{36}$$
$$= \frac{1}{6}$$

c. There are 30 points on the grid that do not represent a sum greater than 9.

$$\therefore P(\text{not D}) = \frac{30}{36}$$
$$= \frac{5}{6}$$

The results of exercises 4 to 6 are the same using a grid or using a table.

8. The sum of the probability that an event, *E*, will occur and the probability that it will not occur is 1, or 100%.

$$P(E)+P(\text{not } E)=1$$

2. Textbook exercises 1, 2, and 3.a. of "Exercises: Checking Your Skills," p. 17

1. $P(\text{pass})+P(\text{fail})=1$

$$\begin{aligned} P(\text{fail}) &= 1-P(\text{pass}) \\ &= 1-\frac{1}{10} \\ &= \frac{9}{10} \end{aligned}$$

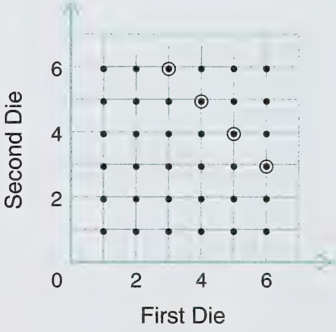
The probability that you will fail a driving test on the first try is $\frac{9}{10}$.

2. **a.** The outcomes in the event "at least 3 eggs are broken" are 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 broken eggs.

b. The outcomes in the event "no more than 2 eggs are broken" are 0, 1, and 2 broken eggs.

c. The complement of the event "at most 1 egg is broken" is "at least 2 eggs are broken" or "more than one egg is broken."

3. **a.** Circle the points on the grid for which the sum of the two numbers is 9.



There are 4 outcomes favourable to the event.

$$\begin{aligned} \therefore\ P(\text{sum of } 9) &= \frac{4}{36} \\ &= \frac{1}{9} \end{aligned}$$

3. a. Textbook exercises 1 and 3 of "Discussing the Ideas," p. 16

1. The fact that the dice are different colours or the same colour does not change the result of the investigation. Instead of calling the dice green and black, you could call them first and second. There are still 36 possible outcomes in the sample space.

3. The probability of an event, A, is determined using the formula $P(A)=\frac{r}{n}$, where r is the number of outcomes favourable to the event and n is the total number of outcomes in the sample space. Since the favourable outcomes are selected from the sample space, $0 \le r \le n$.

If there are no favourable outcomes,

$$\begin{aligned} r=0 \text{ and } P(A) &= \frac{r}{n} \\ &= \frac{0}{n} \\ &= 0 \end{aligned}$$

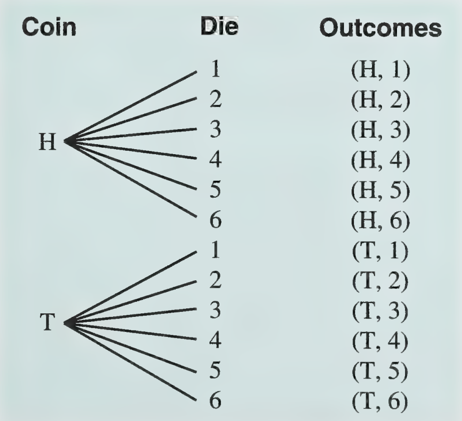
If all the outcomes in the sample space are favourable,

$$\begin{aligned} r=n \text{ and } P(A) &= \frac{r}{n} \\ &= \frac{n}{n} \\ &= 1 \end{aligned}$$

Therefore, the probability that any event, A, will occur must be between 0 and 1 inclusive.

b. Textbook exercises 4, 5, 7, and 8 of "Exercises: Checking Your Skills," pp. 17 and 18

4. a. Draw a tree diagram to list the outcomes in the sample space.



b. Let *A* be the following event:

"The coin lands heads and the die shows an even number."

There are 3 outcomes in event *A*: (H, 2), (H, 4), and (H, 6).

$$\begin{aligned}\therefore\ P(A) &= \frac{3}{12}\\ &= \frac{1}{4}\end{aligned}$$

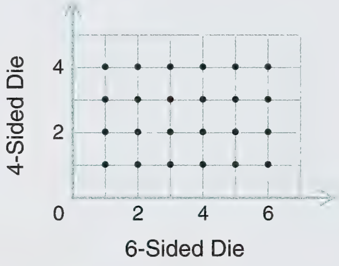
c. Let *B* be the following event:

"The coin lands tails and the die shows a number greater than 2."

There are 4 outcomes in event *B*: (T, 3), (T, 4), (T, 5), and (T, 6).

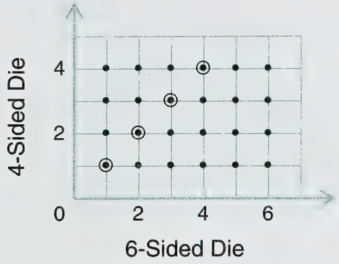
$$\begin{aligned}\therefore\ P(B) &= \frac{4}{12}\\ &= \frac{1}{3}\end{aligned}$$

5. a. Use a grid to display the sample space.



There are 24 outcomes in the sample space.

b. Circle all the outcomes in the sample space that represent throwing doubles. Call this event *E*.

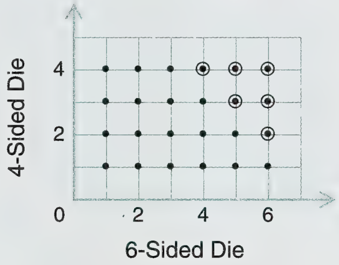


$$\therefore P(E)=\frac{4}{24}$$
$$=\frac{1}{6}$$

c. There are $24-4=20$ outcomes that do not represent doubles.

$$\therefore P(\bar{E})=\frac{20}{24} \quad \text{or} \quad P(\bar{E})=1-P(E)$$
$$=\frac{5}{6} \qquad\qquad =1-\frac{1}{6}$$
$$=\frac{5}{6}$$

d. Circle all the outcomes in *A*.



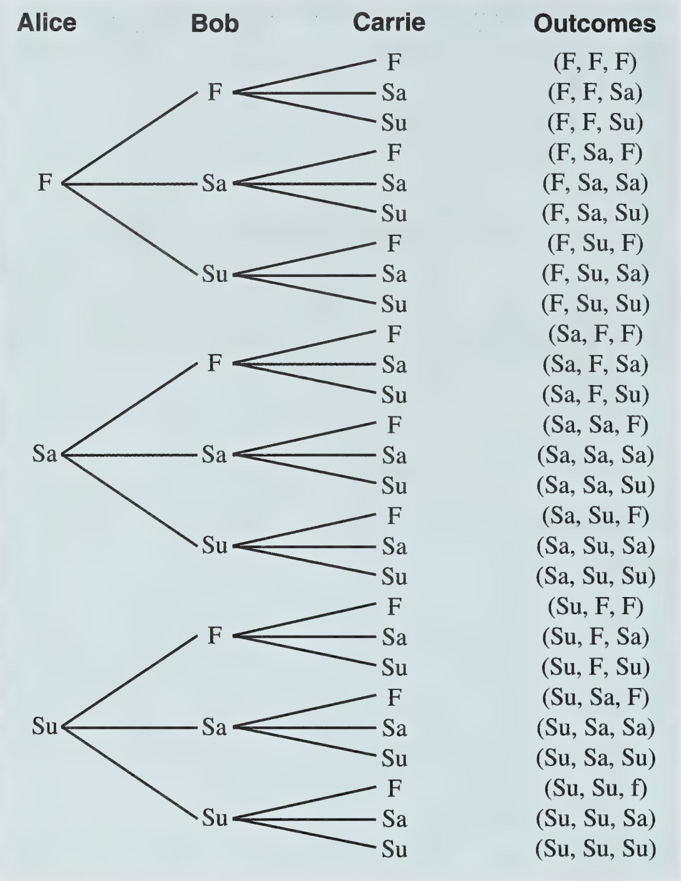
$$\therefore P(A)=\frac{6}{24}$$
$$=\frac{1}{4}$$

e. The complement of *A* is throwing the dice and obtaining a sum of 7 or less.

f. $P(\bar{A})=1-P(A)$
$$=1-\frac{1}{4}$$
$$=\frac{3}{4}$$

Activity 2 (continued)

7. **a.** Use a tree diagram to list the outcomes in the sample space.



Outcomes

(F, F, F)
(F, F, Sa)
(F, F, Su)
(F, Sa, F)
(F, Sa, Sa)
(F, Sa, Su)
(F, Su, F)
(F, Su, Sa)
(F, Su, Su)
(Sa, F, F)
(Sa, F, Sa)
(Sa, F, Su)
(Sa, Sa, F)
(Sa, Sa, Sa)
(Sa, Sa, Su)
(Sa, Su, F)
(Sa, Su, Sa)
(Sa, Su, Su)
(Su, F, F)
(Su, F, Sa)
(Su, F, Su)
(Su, Sa, F)
(Su, Sa, Sa)
(Su, Sa, Su)
(Su, Su, f)
(Su, Su, Sa)
(Su, Su, Su)

b. There are 27 outcomes in the sample space.

c. Let *A* be the following event:

> "All three students completed the assignment on the same evening."

There are 3 outcomes in *A*: (F, F, F), (Sa, Sa, Sa), and (Su, Su, Su).

$$\therefore P(A) = \frac{3}{27}$$

$$= \frac{1}{9}$$

d. Let B be the following event:

> "At least one student did not complete the assignment until Sunday."

Method 1

List the outcomes in B.

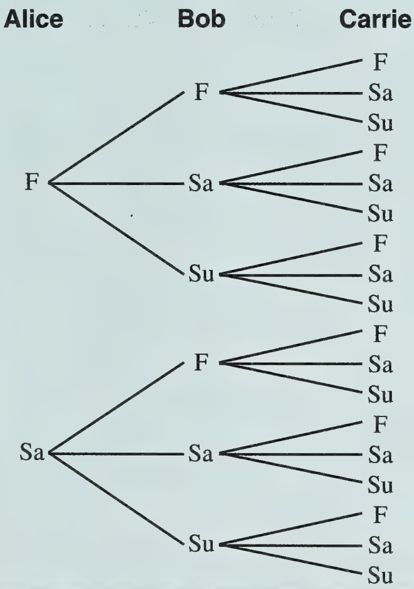
(F, F, Su), (F, Sa, sU), (F, Su, F), (F, Su, Sa), (F, Su, Su), (Sa, F, Su), (Sa, Sa, Su), (Sa, Su, F), (Sa, Su, Sa), (Sa, Su, Su), (Su, F, F,) (Su, F, Sa), (Su, F, Su), (Su, Sa, F), (Su, Sa, Sa), (Su, Sa, Su), (Su, Su, F), (Su, Su, Sa), (Su, Su, Su)

There are 19 outcomes in B.

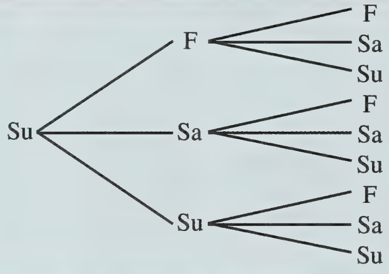
$$\therefore P(B) = \frac{19}{27}$$

Method 2

Add a column to the tree diagram to mark the items with at least one Sunday. Then count these items and proceed as in Method 1.



Outcomes	At Least 1 Sunday
(F, F, F)	
(F, F, Sa)	
(F, F, Su)	✓
(F, Sa, F)	
(F, Sa, Sa)	
(F, Sa, Su)	✓
(F, Su, F)	✓
(F, Su, Sa)	✓
(F, Su, Su)	✓
(Sa, F, F)	
(Sa, F, Sa)	
(Sa, F, Su)	✓
(Sa, Sa, F)	
(Sa, Sa, Sa)	
(Sa, Sa, Su)	✓
(Sa, Su, F)	✓
(Sa, Su, Sa)	✓
(Sa, Su, Su)	✓

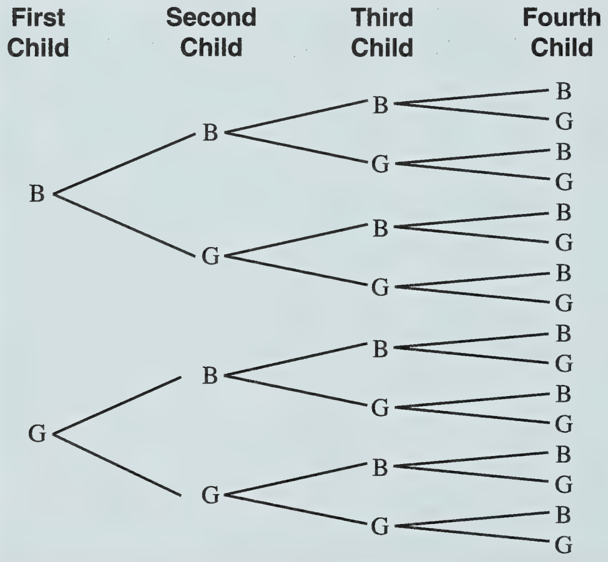


(Su, F, F) ✓
(Su, F, Sa) ✓
(Su, F, Su) ✓
(Su, Sa, F) ✓
(Su, Sa, Sa) ✓
(Su, Sa, Su) ✓
(Su, Su, F) ✓
(Su, Su, Sa) ✓
(Su, Su, Su) ✓

There are 19 outcomes in *B*.

$$\therefore P(B)=\frac{19}{27}$$

8. **a.** Use a tree diagram to list the outcomes in the sample space.



Outcomes

(B, B, B, B)
(B, B, B, G)
(B, B, G, B)
(B, B, G, G)
(B, G, B, B)
(B, G, B, G)
(B, G, G, B)
(B, G, G, G)
(G, B, B, B)
(G, B, B, G)
(G, B, G, B)
(G, B, G, G)
(G, G, B, B)
(G, G, B, G)
(G, G, G, B)
(G, G, G, G)

There are 16 outcomes in the sample space.

b. There is only 1 outcome for which all children are girls: (G, G, G, G).

$$P(\text{all girls}) = \frac{1}{16}$$

c. Let B be the following event:

"All 4 children are boys."

There is only 1 outcome in B: (B, B, B, B).

$$\therefore\ P(B) = \frac{1}{16}$$

Determine the complement of B, that is at least one child is a girl.

$$\begin{aligned} P(\bar{B}) &= 1 - P(B) \\ &= 1 - \frac{1}{16} \\ &= \frac{15}{16} \end{aligned}$$

The probability that the couple will have at least one girl is $\frac{15}{16}$.

d. Let A be the following event:

"The couple has two boys and two girls."

List the outcomes in A.

(B, B, G, G), (B, G, B, G), (B, G, G, B), (G, B, B, G), (G, B, G, B), (G, G, B, B)

There are 6 outcomes in A.

$$\begin{aligned} \therefore\ P(A) &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

The probability that the couple will have two boys and two girls is $\frac{3}{8}$.

c. **Textbook exercise 10 of "Exercises: Extending Your Thinking," p. 19**

10. **a.** There are 2 ways of answering each question. Therefore, in a tree diagram representing 5 questions, there would be $2 \times 2 \times 2 \times 2 \times 2 = 32$ outcomes.

b. There is only 1 outcome representing a score of 100%.

$$\therefore\ P(100\%) = \frac{1}{32}$$

c. Let *W* be the following event:

"All questions are answered incorrectly."

There is only 1 outcome representing all incorrect answers.

$$\therefore\ P(W) = \frac{1}{32}$$

Form the complement of *W*.

"At least one question is correct."

$$\begin{aligned}\therefore\ P(\bar{W}) &= 1 - P(W) \\ &= 1 - \frac{1}{32} \\ &= \frac{31}{32}\end{aligned}$$

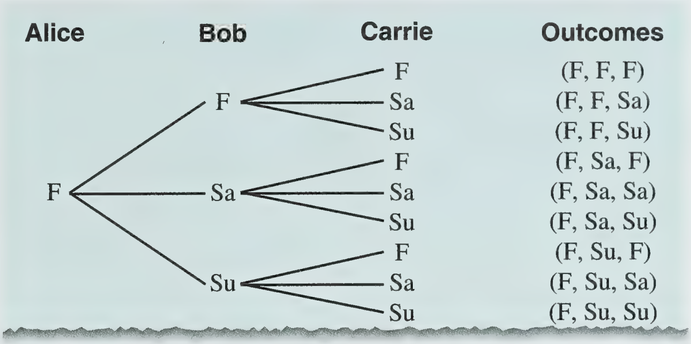
The probability that the student will get at least one question correct is $\frac{31}{32}$.

4. **Textbook exercise "Communicating the Ideas," p. 19**

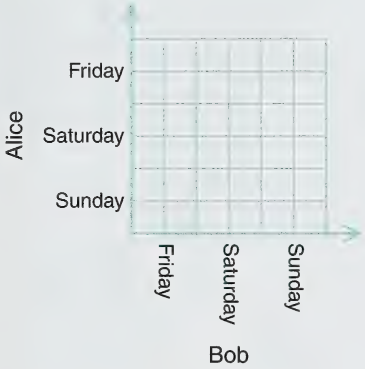
Answers will vary. A sample answer is given.

It is true; tree diagrams can be used for finding all sample spaces, but tables and grids cannot. Grids and tables only work well for two-dimensional sample spaces.

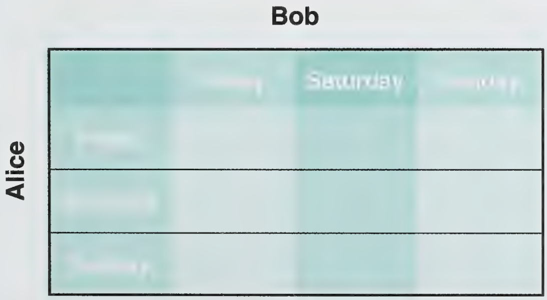
Recall the tree diagram in the answer to exercise 7 of “Exercises: Checking Your Skills” on page 18 of the textbook, where three students completed their assignment either on Friday, Saturday, or Sunday.



To create a sample space for this three-dimensional problem using a grid would be very difficult because grids are inherently two-dimensional. Once you have placed Alice on one axis of the grid and Bob on the other, where would Carrie be placed?



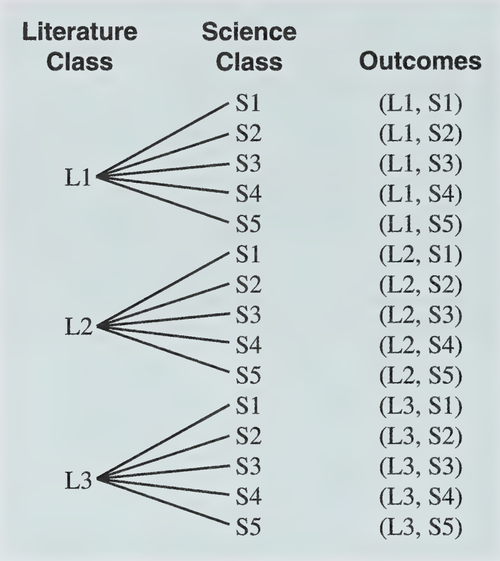
Using a table to create this sample space would also be difficult because, like grids, tables are two-dimensional. Once Alice is placed on the left side and Bob is placed on the top, there is no location for Carrie.


Bob
Alice
Saturday

Activity 3: The Fundamental Counting Principle

1. Textbook exercises 1 to 4 of "Investigation: Counting Without Counting," p. 20

1. a. Use a tree diagram to list the outcomes of the sample space.



b. The requirements of one literature class and one science class can be satisfied in 15 different ways, since there are 15 different outcomes.

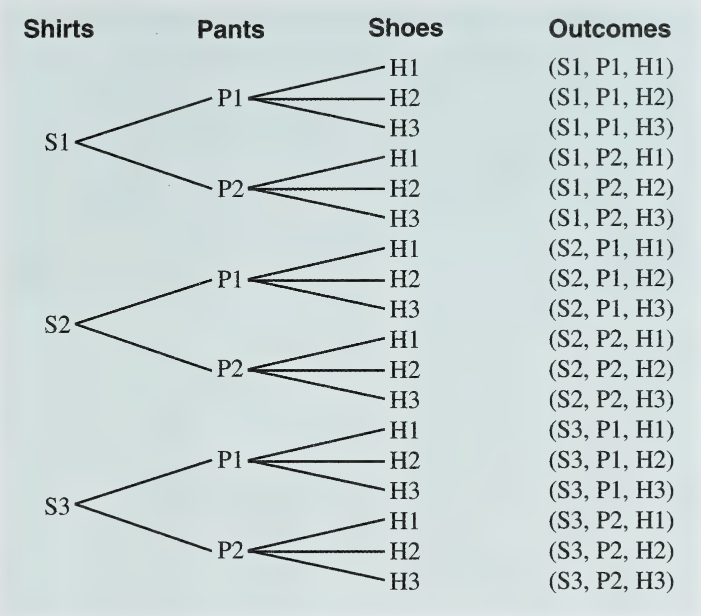
c. A student can choose a literature class in 3 different ways.

d. A student can choose a science class in 5 different ways.

e. You can determine the number of ways of selecting one literature class and one science class by multiplying the number of ways of selecting one literature class by the number of ways of selecting one science class.

$$\begin{aligned}\therefore \text{ Number of ways} &= 3 \times 5 \\ &= 15\end{aligned}$$

2. **a.** Use a tree diagram to list the outcomes in the sample space.



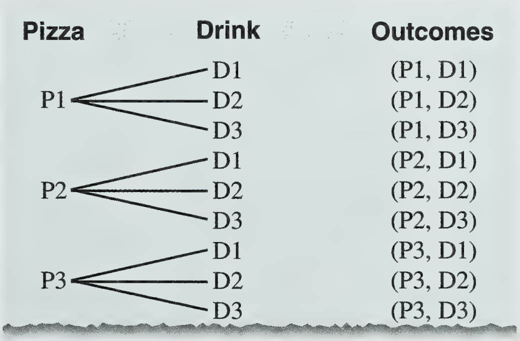
b. Joe can get dressed 18 different ways in the morning.

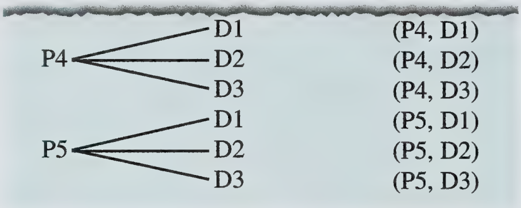
c. You can determine the number of different outfits Joe can select by multiplying the number of ways he can choose a shirt by the number of ways he can choose a pair of pants by the number of ways he can choose a pair of shoes.

$$\therefore \text{ Number of ways} = 3 \times 2 \times 3$$
$$= 18$$

3. **a.** There should be $5 \times 3 = 15$ ways of selecting a pizza and a drink.

b. Use a tree diagram to verify that there are 15 outcomes in the sample space.





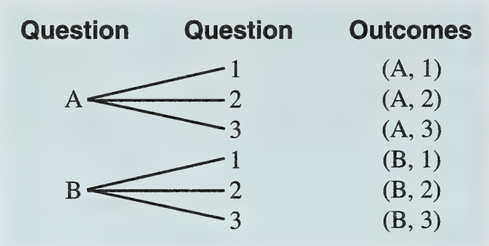
4. If one item can be selected in *m* different ways and a second item in *n* different ways, the two items can be selected in $m \times n$ ways.

2. a. **Textbook exercises 1 and 3 of "Discussing the Ideas," p. 23**

1. Answers will vary. A sample answer is given.

Suppose on a math exam, a student must answer one of questions A and B and one of questions 1, 2, and 3. How many ways can the student make the selections?

Use a tree diagram to list the choices.



There are $2 \times 3 = 6$ choices. The number of ways the items can be selected must be multiplied, not added.

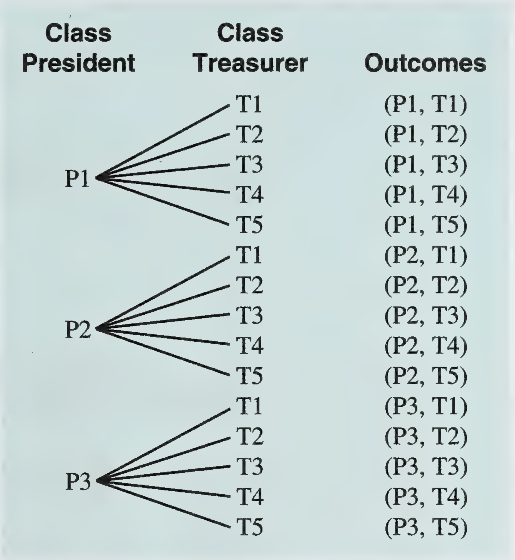
3. Your friend made the mistake of adding the ways the items can be answered instead of multiplying them. The correct number of ways is calculated as follows:

$$\begin{aligned}\text{Number of ways} &= 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\\ &= 2^{10}\\ &= 1024\end{aligned}$$

There are 1024 ways of completing the exam.

b. Textbook exercises 1, 2, and 3 of "Exercises: Checking Your Skills," p. 23

1. a. Use a tree diagram to list the outcomes of the sample space.



There are 15 outcomes possible.

b. Use the Fundamental Counting Principle.

There are 3 candidates for president.
There are 5 candidates for treasurer.

Therefore, there are $3 \times 5 = 15$ possible outcomes of the election. This matches the answer to exercise 1.a.

2. A card can be selected 52 different ways.
A die can turn up 6 different ways.
A coin can turn up 2 different ways.

Therefore, there are $52 \times 6 \times 2 = 624$ possible results.

3. a. There are 5 ways of selecting a blouse.
There are 4 ways of selecting a skirt.
There are 4 ways of selecting a sweater.

Using the Fundamental Counting Principle, there are $5 \times 4 \times 4 = 80$ outfits possible.

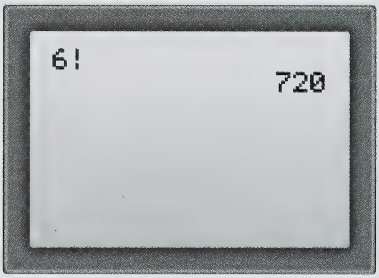
Activity 3 (continued)

b. If the outfit includes a sweater, from exercise 3.a., there are 80 possibilities. If the outfit does not include a sweater, there are only $5 \times 4 = 20$ possibilities.

Therefore, if the outfit may or may not include a sweater, there are $80 + 20 = 100$ possible outfits.

3. a. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

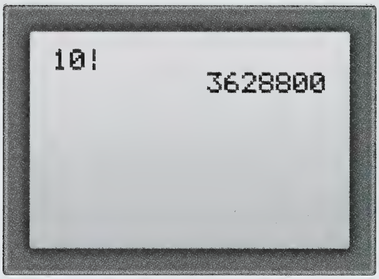
$= 720$



```
6!
                720
```

b. $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3\,628\,800$



```
10!
            3628800
```

c. The expression is undefined. The number in the factorial must be a whole number.

Note: The TI-83 and TI-83 Plus graphing calculators will return the value 0.886 226 925 5 when you enter $(0.5)!$. Mathematicians like to find ways to generalize facts they learn. The idea of factorial, which only makes sense for whole numbers, has been extended. A function known as the gamma function $(\Gamma(x))$ has been defined. For a whole number, it gives the same value as the factorial, but it also gives values for positive real numbers. The calculator is actually giving back $\Gamma\left(\frac{1}{2}\right)$, not $(0.5)!$. Remember that $\frac{1}{2}!$ or 0.5! is not defined.

This may occur with other graphing calculators as well.

d. The expression is undefined. The number in the factorial must be a positive whole number.

4. a. Textbook questions 5 to 10 of "Exercises: Checking Your Skills," pp. 24 and 25

5. a. There are 3 different buses from Jai's home to the downtown train station.
There are 2 different buses from the downtown train station to work.

Therefore, there are $3 \times 2 = 6$ different ways Jai can take only buses to get to work.

b. There are 2 different trains from Jai's home to the downtown train station.
There are 4 different trains from the downtown train station to work.

Therefore, there are $2 \times 4 = 8$ different ways Jai can take only trains to get to work.

c. There are 5 ways Jai can go to the downtown train station from his home.
There are 6 ways Jai can go to work from the downtown train station.

Therefore, there are $5 \times 6 = 30$ different ways Jai can get to work.

6. a. There are 26 different letters and 10 different numbers. Assume that letters and numbers may be repeated.

26	26	26
first letter	second letter	third letter
10	10	10
first number	second number	third number

$\therefore\ 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17\ 576\ 000$

There are 17 576 000 different licence plates possible.

b.

26	25	24
first letter	second letter	third letter
10	10	10
first number	second number	third number

$\therefore\ 26 \times 25 \times 24 \times 10 \times 10 \times 10 = 15\ 600\ 000$

There are 15 600 000 different licence plates possible if no letter is repeated.

c.

26	26	26
first letter	second letter	third letter
10	9	8
first number	second number	third number

$\therefore\ 26 \times 26 \times 26 \times 10 \times 9 \times 8 = 12\ 654\ 720$

There are 12 654 720 different licence plates possible if no number is repeated.

d. First find the probability that no letter is repeated. Call this event *A*.

$$
\begin{aligned}
P(A) &= \frac{\text{Number of plates with no letter repeated}}{\text{Total number of plates}} \\
&= \frac{15\ 600\ 000}{17\ 576\ 000} \\
&\doteq 0.888 \text{ or } 88.8\%
\end{aligned}
$$

Form the complement of event *A*.

"At least one letter is repeated."

$$
\begin{aligned}
\therefore\ P(\bar{A}) &= 1 - P(A) \\
&\doteq 1 - 0.888 \\
&\doteq 0.112 \text{ or } 11.2\%
\end{aligned}
$$

The probability that you will receive a licence plate with at least one letter repeated is approximately 11.2%.

e. First find the probability that no digit is repeated. Call this event *B*.

$$
\begin{aligned}
P(B) &= \frac{\text{Number of plates with no digit repeated}}{\text{Total number of plates}} \\
&= \frac{12\ 654\ 720}{17\ 576\ 000} \\
&= 0.72 \text{ or } 72\%
\end{aligned}
$$

Form the complement of event *B*.

"At least one digit is repeated."

$$
\begin{aligned}
\therefore\ P(\bar{B}) &= 1 - P(B) \\
&= 1 - 0.72 \\
&= 0.28 \text{ or } 28\%
\end{aligned}
$$

The probability that you will receive a licence plate with at least one digit repeated is 28%.

7. **a.** At the end of the second period, the winning team's score could be 0, 1, 2, 3, 4, or 5. Likewise, the losing team's score could be 0, 1, or 2.

Therefore, there are $6\times 3=18$ different scores possible at the end of the second period.

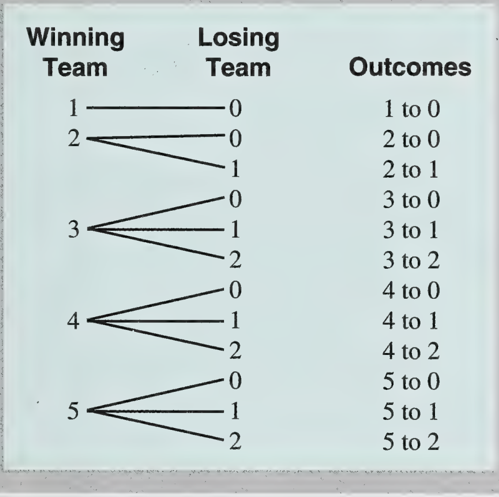
b. If both teams had scored at least one goal, the winning team's score at the end of the second period could be 1, 2, 3, 4, or 5. Likewise, the losing team's score could be 1 or 2.

Therefore, there are $5\times 2=10$ different scores possible at the end of the second period.

c. If the winning team was in the lead, the scores could have been 1 to 0, 2 to 0, 2 to 1, 3 to 0, 3 to 1, 3 to 2, 4 to 0, 4 to 1, 4 to 2, 5 to 0, 5 to 1, or 5 to 2.

There are 12 different scores possible at the end of the second period.

Note: These outcomes could have been found using a tree diagram.



8. **a.** $60\times 60\times 60=216\ 000$

With no restrictions, there are 216 000 combinations possible.

b. $60\times 59\times 58=205\ 320$

If no number is repeated, there are 205 320 combinations possible.

c. There are 60 numbers from 0 to 59. Half of these numbers are odd. Therefore, there are 30 odd numbers.

If odd numbers can be repeated, there are $30 \times 30 \times 30 = 27\ 000$ combinations possible.

Let A be the following event:

"All three numbers are odd."

$$\begin{aligned} \therefore\ P(A) &= \frac{\text{Number of combinations with only odd numbers}}{\text{Total number of combinations}} \\ &= \frac{27\ 000}{216\ 000} \\ &= 0.125 \text{ or } 12.5\% \end{aligned}$$

The probability that all three numbers of the combination are odd is 0.125 or 12.5%.

d. The prime numbers from 0 to 59 are as follows:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, and 59

There are 17 prime numbers.

If numbers can be repeated, there are $17 \times 17 \times 17 = 4913$ combinations possible.

Let B be the following event:

"All three numbers are prime."

$$\begin{aligned} \therefore\ P(B) &= \frac{\text{Number of combinations with only prime numbers}}{\text{Total number of combinations}} \\ &= \frac{4913}{216\ 000} \\ &\doteq 0.023 \text{ or } 2.3\% \end{aligned}$$

The probability that all three numbers are prime is about 0.023 or 2.3%.

9. a. Each question can be answered 4 different ways.

b. Because there are 5 questions, there are $4 \times 4 \times 4 \times 4 \times 4 = 1024$ different ways of completing the test.

c. There is only 1 way to answer all of the questions correctly. Therefore, the probability of answering the test correctly is $\frac{1}{1024}$ or about 0.1%.

10. **a.** There are $5\times10=50$ positions on the card. Each position can be punched with a hole or left blank. Therefore, there are $2^{50}\doteq1.13\times10^{15}$ different keys possible.

b. Divide the number of possible keys by the number of people on Earth.

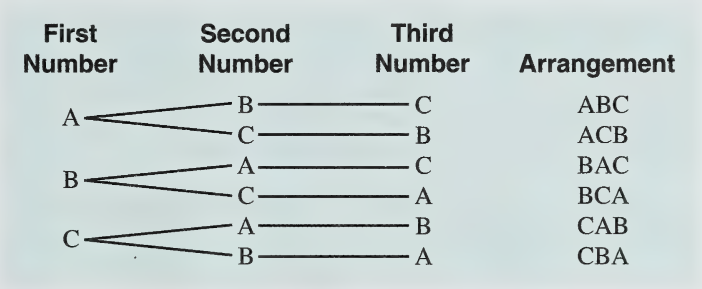
$$\begin{aligned}\text{Number of keys per person} &= \frac{2^{50}}{6\ 000\ 000\ 000}\\ &\doteq 187\ 650\end{aligned}$$

There would be about 187 650 keys for each person.

b. Textbook exercise 11 of "Exercises: Extending Your Thinking," p. 25

11. There are $10\times10\times10=1000$ ways of selecting the numbers for Straight Play. There is only 1 possible way of winning. Therefore, the probability of winning is $\frac{1}{1000}$.

For Box Play, any order is possible. Let the winning numbers be A, B, and C. Use a tree diagram to determine all possible arrangements.



There are 6 possible winning arrangements. Therefore, the probability of winning in Box Play is $\frac{6}{1000}$, or about $\frac{1}{167}$.

5. **a. Project Book exercise 1 of "Getting Started," p. 138**

1. If you want to minimize cost, cut the pieces from the 48-inch by 96-inch sheet. **Note:** When a saw blade cuts material, some of the material's width is lost; we will ignore this in this exercise. There is some interesting information on this topic on the following website:

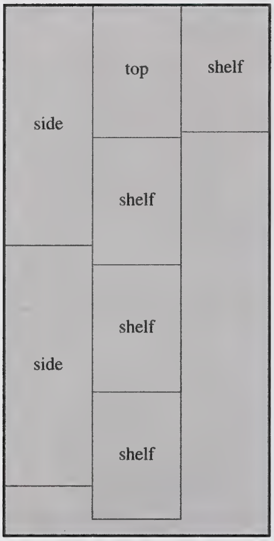
http://www.borealforest.org/world/innova/processing.htm

Since the top, sides, and shelves are made from $\frac{1}{2}$-inch thick melamine, the following are the measurements of each part of the shelf unit that needs to be cut out of the 48-inch by 96-inch sheet.

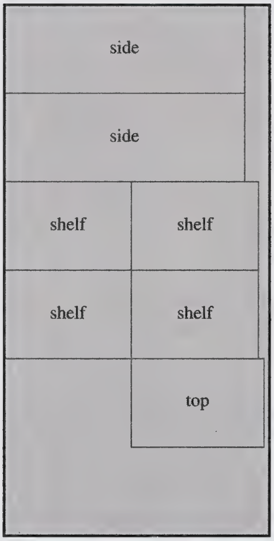
Top: 24 inches by 16 inches ← given
Each side: 16 inches by 43.5 inches ← The top is $\frac{1}{2}$-inch thick.
Each shelf: 16 inches by 23 inches ← Each side is $\frac{1}{2}$-inch thick.

Here are three possible ways to cut the large sheet into the sizes required for the shelf unit.

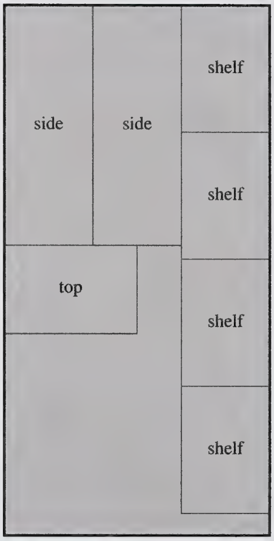
Plan 1



Plan 2



Plan 3



	Amount of Melamine Edging (inches)		
Part	Plan 1	Plan 2	Plan 3
Top (front, both sides)	24 + 16 + 16	24 + 16 + 16	24 + 16 + 16
Shelf 1 (front only)	23	23	23
Shelf 2 (front only)	23	23	23
Shelf 3 (front only)	23	23	23
Shelf 4 (front only)	23	23	23
Side 1 (front only)	43.5	43.5	43.5
Side 2 (front only)	43.5	43.5	43.5
Total	235	235	235

Each plan requires 235 inches of melamine edging.

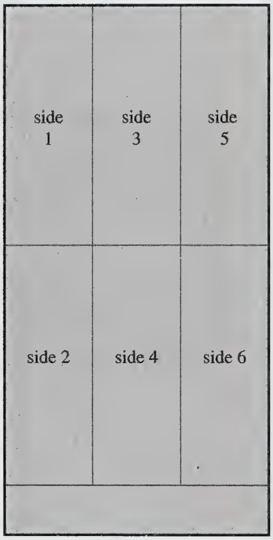
$$\begin{aligned}\therefore \text{ Cost of edging} &= 235 \text{ inches} \times \frac{\$0.15}{12 \text{ inches}}\\ &= \$2.94\end{aligned}$$

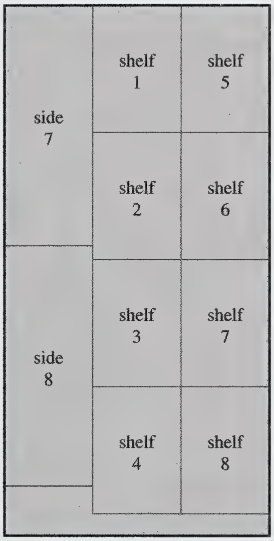
$$\begin{aligned}\therefore \text{ Total cost} &= \text{Cost of melamine} + \text{Cost of edging}\\ &= \$20.00 + \$2.94\\ &= \$22.94\end{aligned}$$

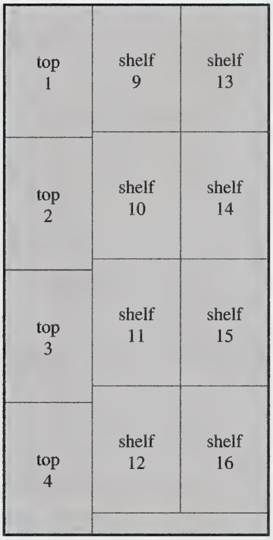
Note: If you used the 16-inch × 96-inch sheets, the added costs would include adding melamine to the 16-inch ends of the top. You will need 3 feet of melamine, since twice 16 inches is more than 2 feet. The total cost, therefore, would be $\$8.00 \times 3 + 3 \times \$0.15 = \$24.45$.

b. Project Book exercise 1 of "Getting Started," p. 138

1. The required pieces for four units can be cut from three 48-inch by 96-inch sheets.







Because there are four units, $4 \times 235 = 940$ inches of melamine edging is needed.

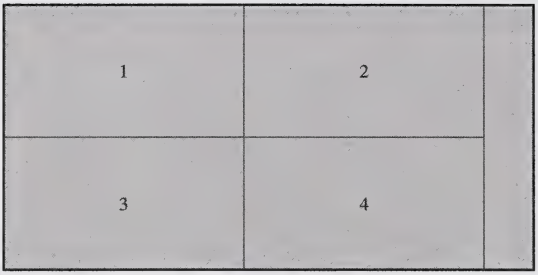
$$\begin{aligned}\text{Cost of edging} &= 940 \text{ inches} \times \frac{\$0.15}{12 \text{ inches}}\\ &= \$11.75\end{aligned}$$

$$\begin{aligned}\therefore \text{ Total cost of 4 units} &= \text{Cost of melamine} + \text{Cost of edging}\\ &= 3 \times \$20.00 + \$11.75\\ &= \$60.00 + \$11.75\\ &= \$71.75\end{aligned}$$

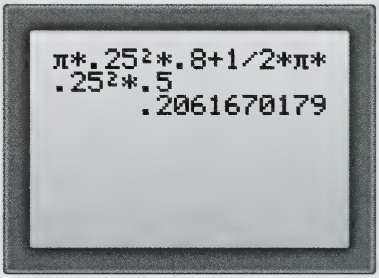
Activity 3 (continued)

c. Project Book exercise 2 of "Getting Started," p. 139

2. You can cut out 4 backs from one sheet of 4-foot by 8-foot hardboard.



6. Project Book exercises 3 and 4 of "Getting Started," p. 139



```
π*.25²*.8+1/2*π*
.25²*.5
           .2061670179
```

3. Volume = Volume of cylinder + Volume of half-cylinder

$$= \pi r^2 h + \frac{1}{2}\pi r^2 h$$

$$= \pi(0.25)^2(0.8) + \frac{1}{2}\pi(0.25)^2(0.5)$$

$$\doteq 0.2062$$

4. Volume of 10 supports $\doteq 10 \times 0.2062\ \text{cm}^3$

$$\doteq 2.062\ \text{cm}^3$$

$$\therefore \text{Total cost} = 2.062\ \text{cm}^3 \times 0.5¢/\text{cm}^3$$

$$= 1¢$$

7. **Textbook exercise "Communicating the Ideas," p. 25**

Answers will vary. A sample answer is given.

Often, when calculating probabilities, the number of outcomes in the event and the number of outcomes in the sample space are determined using the Fundamental Counting Principle.

For example, suppose a die is tossed twice. What is the probability that an even number appears on the first toss and an odd number appears on the second toss?

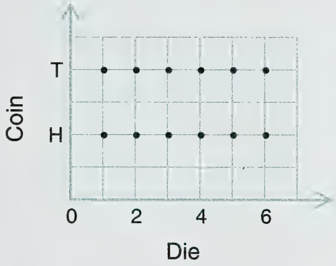
The die can turn up 6 different ways in each toss. Using the Fundamental Counting Principle, there are $6 \times 6 = 36$ outcomes in the sample space. There are 3 even numbers and 3 odd numbers on the die. Using the Fundamental Counting Principle, there are $3 \times 3 = 9$ favourable outcomes in the event.

$$\begin{aligned}\therefore P(\text{even than odd}) &= \frac{9}{36}\\ &= \frac{1}{4}\end{aligned}$$

Activity 4: Independent and Dependent Events

1. **Textbook exercises 1 to 4 of "Investigation 1: 'AND' Probabilities," p. 28**

1.



There are 12 outcomes in the sample space.

2. **a.** $P(\text{A}) = \frac{1}{2}$

b. $P(\text{B}) = \frac{1}{6}$

c. There is only 1 outcome in A and B: $(\text{H}, 6)$.

$\therefore P(\text{A and B}) = \frac{1}{12}$

d. $\frac{1}{12} = \frac{1}{2} \times \frac{1}{6}$

3. **a.** $P(\text{M}) = \frac{1}{2}$

   b. $$\begin{aligned} P(\text{N}) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

   c. There are 3 outcomes in M and N: (T, 1), (T, 3), and (T, 5).

   $$\begin{aligned} \therefore\ P(\text{M and N}) &= \frac{3}{12} \\ &= \frac{1}{4} \end{aligned}$$

   d. $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$

4. **a.** $P(\text{S}) = \frac{1}{2}$

   b. $$\begin{aligned} P(\text{T}) &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

   c. There are 4 outcomes in S and T: (H, 1), (H, 2), (H, 3), and (H, 4).

   $$\begin{aligned} \therefore\ P(\text{S and T}) &= \frac{4}{12} \\ &= \frac{1}{3} \end{aligned}$$

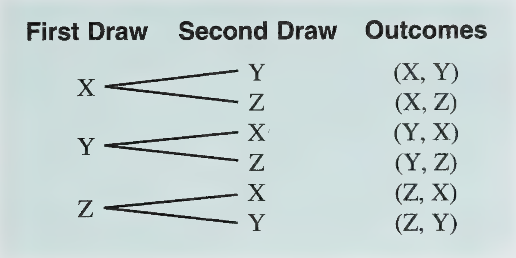
   d. $\frac{1}{3} = \frac{1}{2} \times \frac{2}{3}$

2. Textbook exercises 1 to 4 of "Investigation 2: More 'AND' Probabilities," p. 29

1. No, these events are not independent. Because the first marble is not replaced before the second is drawn, the probability of drawing a certain marble on the second draw is affected by the occurrence of the first draw.

2. There are 3 choices for the first draw and 2 choices for the second draw. Therefore, using the Fundamental Counting Principle, there are $3 \times 2 = 6$ possible outcomes.

3.



4. **a.** There are 4 outcomes in A: (X, Y), (X, Z), (Y, X), and (Z, X).

$$\begin{aligned}\therefore P(\text{A}) &= \frac{4}{6}\\ &= \frac{2}{3}\end{aligned}$$

b. There are 4 outcomes in B: (X, Y), (Y, X), (Y, Z), and (Z, Y).

$$\begin{aligned}\therefore P(\text{B}) &= \frac{4}{6}\\ &= \frac{2}{3}\end{aligned}$$

c. There are 2 outcomes in A and B: (X, Y) and (Y, X).

$$\begin{aligned}\therefore P(\text{A and B}) &= \frac{2}{6}\\ &= \frac{1}{3}\end{aligned}$$

d.

LS	RS
$P(\text{A and B})$	$P(\text{A})\times P(\text{B})$
$=\frac{1}{3}$	$=\frac{2}{3}\times\frac{2}{3}$
	$=\frac{4}{9}$

LS $\neq$ RS

For these events, $P(\text{A and B}) \neq P(\text{A})\times P(\text{B})$.

Activity 4 (continued)

3. **a. Textbook exercises 1 and 3 of "Discussing the Ideas," p. 31**

   1. If the first marble was replaced before the second is drawn, the events would be independent.

   3. The statement is false. The probability of heads and tails are the same for each toss, regardless of how many consecutive heads have occurred. The events in tossing a coin are independent.

   b. Textbook exercise 1 of "Exercises: Checking Your Skills," p. 31

   1. a. The toss of a coin and the roll of a die are independent events.

   b. These events are dependent because the first card is not replaced. If the first card drawn is an ace, there would only be 3 aces available for the second draw.

   c. These events are independent because the first card is replaced. Even if a queen were drawn first, there could still be 4 queens available for the second draw.

   d. The successive rolls of a die are independent events.

   e. These events are dependent. As students are selected, your chances improve, because the number of students available decreases.

   f. Two successive games of solitaire are independent events.

4. **a. Textbook exercises 2 to 9 of "Exercises: Checking Your Skills," pp. 31 and 32**

   2. Let A and B represent the following events:

   A: "A young driver takes driver training."
   B: "A young driver has an accident in his or her first year of driving."

   $$P(A) = 60\% = 0.60 \qquad P(B) = 25\% = 0.25 \qquad P(A \text{ and } B) = 10\% = 0.10$$

   If A and B are independent events, $P(A \text{ and } B) = P(A) \times P(B)$. Check.

LS	RS
$P(A \text{ and } B)$	$P(A) \times P(B)$
$= 0.10$	$= 0.60 \times 0.25$
	$= 0.15$

   $$\text{LS} \neq \text{RS}$$

   Because $P(A \text{ and } B) \neq P(A) \times P(B)$, events A and B are not independent.

3. Let A and B represent the following events:

A: "A grade 12 student enrols in a tutorial program."
B: "A grade 12 student finds the academic demands tougher than in grade 11."

$$P(A) = 40\% \quad = 0.40 \qquad P(B) = 50\% \quad = 0.50 \qquad P(A \text{ and } B) = 20\% \quad = 0.20$$

If A and B are independent events, $P(A \text{ and } B) = P(A) \times P(B)$. Check.

LS	RS
$P(A \text{ and } B)$	$P(A) \times P(B)$
$= 0.20$	$= 0.40 \times 0.50$
	$= 0.20$

LS = RS

Because $P(A \text{ and } B) = P(A) \times P(B)$, events A and B are independent.

4. **a.** Let A and B represent the following events:

A: "The first card drawn is a diamond."
B: "The second card drawn is a diamond."

Because the first card is replaced before the second card is drawn, A and B are independent events.

There are 13 diamonds in a standard deck.

$$\begin{aligned} \therefore\ P(A) = P(B) &= \frac{13}{52} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \therefore\ P(A \text{ and } B) &= P(A) \times P(B) \\ &= \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{16} \end{aligned}$$

The probability of drawing two diamonds is $\frac{1}{16}$.

b. Let A and B represent the following events:

A: "A red card is drawn first."
B: "A red card is drawn second."

Events A and B are independent.

There are 26 red cards in a standard deck.

$$\begin{aligned}\therefore\ P(A)=P(B)&=\frac{26}{52}\\&=\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\therefore\ P(A\ \text{and}\ B)&=P(A)\times P(B)\\&=\frac{1}{2}\times\frac{1}{2}\\&=\frac{1}{4}\end{aligned}$$

The probability of drawing two red cards is $\frac{1}{4}$.

c. Let A and B represent the following events:

A: "A face card is drawn first."
B: "A face card is drawn second."

Events A and B are independent.

There are 12 face cards in a standard deck.

$$\begin{aligned}\therefore\ P(A)=P(B)&=\frac{12}{52}\\&=\frac{3}{13}\end{aligned}$$

$$\begin{aligned}\therefore\ P(A\ \text{and}\ B)&=P(A)\times P(B)\\&=\frac{3}{13}\times\frac{3}{13}\\&=\frac{9}{169}\end{aligned}$$

The probability of drawing two face cards is $\frac{9}{169}$.

d. Let *A* and *B* represent the following events:

A: "A 5 is drawn first."
B: "A 5 is drawn second."

Events *A* and *B* are independent.

There are four 5s in a standard deck.

$$\begin{aligned}\therefore\ P(A)=P(B)&=\frac{4}{52}\\&=\frac{1}{13}\end{aligned}$$

$$\begin{aligned}\therefore\ P(A \text{ and } B)&=P(A)\times P(B)\\&=\frac{1}{13}\times\frac{1}{13}\\&=\frac{1}{169}\end{aligned}$$

The probability of drawing two 5s is $\frac{1}{169}$.

e. Let *A* and *B* be the following events:

A: "The first card is not a club."
B: "The second card is not a club."

Events *A* and *B* are independent events.

In a standard deck, there are 13 clubs and $52-13=39$ other cards.

$$\begin{aligned}\therefore\ P(A)=P(B)&=\frac{39}{52}\\&=\frac{3}{4}\end{aligned}$$

$$\begin{aligned}\therefore\ P(A \text{ and } B)&=P(A)\times P(B)\\&=\frac{3}{4}\times\frac{3}{4}\\&=\frac{9}{16}\end{aligned}$$

The probability that neither card is a club is $\frac{9}{16}$.

5. **a.** $P(\text{G}) = \frac{1}{2}$

$$\begin{aligned}\therefore P(4\text{G}) &= P(\text{G}) \times P(\text{G}) \times P(\text{G}) \times P(\text{G}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16}\end{aligned}$$

The probability of having four girls is $\frac{1}{16}$.

b. $P(\text{B}) = P(\text{G}) = \frac{1}{2}$

$$\begin{aligned}\therefore P(2\text{B then } 2\text{G}) &= P(\text{B}) \times P(\text{B}) \times P(\text{G}) \times P(\text{G}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16}\end{aligned}$$

The probability of having two boys and then two girls is $\frac{1}{16}$.

c. Let *A* be the following event:

A: “All 4 children are boys.”

$$\begin{aligned}\therefore P(A) &= P(\text{B}) \times P(\text{B}) \times P(\text{B}) \times P(\text{B}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16}\end{aligned}$$

Form the complement of *A*.

$\bar{A}$: “At least one child is a girl.”

$$\begin{aligned}\therefore P(\bar{A}) &= 1 - P(A) \\ &= 1 - \frac{1}{16} \\ &= \frac{15}{16}\end{aligned}$$

The probability of having at least one girl is $\frac{15}{16}$.

6. If the first ball drawn is white, there will be 2 white balls and 2 black balls left because the ball is not replaced.

$$\begin{aligned}\therefore\ P(\text{white}) &= \frac{2}{4}\\ &= \frac{1}{2}\end{aligned}$$

The probability the second ball is also white is $\frac{1}{2}$.

7. a. Let *M* and *F* represent the following events:

M: "The student completes her math assignment."
F: "The student completes her French assignment."

$$\therefore\ P(M) = 0.6 \text{ and } P(F) = 0.4$$

Because the events are independent,

$$\begin{aligned}P(M \text{ and } F) &= P(M) \times P(F)\\ &= 0.6 \times 0.4\\ &= 0.24\end{aligned}$$

The probability that the student completes both assignments is 0.24.

b. Form the complements of events *M* and *F*.

$\bar{M}$: "The student does not complete the math assignment."
$\bar{F}$: "The student does not complete the French assignment."

$$\begin{aligned}P(\bar{M}) &= 1 - P(M) \\ &= 1 - 0.6 \\ &= 0.4\end{aligned} \qquad \text{and} \qquad \begin{aligned}P(\bar{F}) &= 1 - P(F) \\ &= 1 - 0.4 \\ &= 0.6\end{aligned}$$

Because the events are independent,

$$\begin{aligned}P(\bar{M} \text{ and } \bar{F}) &= P(\bar{M}) \times P(\bar{F})\\ &= 0.4 \times 0.6\\ &= 0.24\end{aligned}$$

The probability that the student completes neither assignment is 0.24.

8. **a.** $P(\text{T}) = \frac{1}{2}$

$$\begin{aligned}\therefore\ P(5\text{T}) &= P(\text{T}) \times P(\text{T}) \times P(\text{T}) \times P(\text{T}) \times P(\text{T}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{32}\end{aligned}$$

The probability that the coin lands tails on 5 successive tosses is $\frac{1}{32}$.

b. The complement of 5 tails is at least 1 head on 5 successive tosses.

$$\begin{aligned}P(\text{at least } 1\text{H}) &= 1 - P(5\text{T}) \\ &= 1 - \frac{1}{32} \\ &= \frac{31}{32}\end{aligned}$$

The probability that the coin will land heads at least once in 5 tosses is $\frac{31}{32}$.

c. First, find the number of tosses required so that the probability of all tails is less than or equal to 0.01 or 1.0%.

$$\begin{aligned}P(5\text{T}) &= \left(\frac{1}{2}\right)^5 \\ &= \frac{1}{32} \\ &\doteq 0.031 \text{ or } 3.1\% \quad \longleftarrow \text{too large}\end{aligned}$$

$$\begin{aligned}P(6\text{T}) &= \left(\frac{1}{2}\right)^6 \\ &= \frac{1}{64} \\ &\doteq 0.016 \text{ or } 1.6\% \quad \longleftarrow \text{too large}\end{aligned}$$

$$\begin{aligned}P(7\text{T}) &= \left(\frac{1}{2}\right)^7 \\ &= \frac{1}{128} \\ &\doteq 0.008 \text{ or } 0.8\% \quad \longleftarrow \text{less than } 1\%\end{aligned}$$

The complement of all tails is at least 1 head on 7 tosses.

$$\begin{aligned}\therefore\ P(\text{at least 1H}) &\doteq 1-0.008\\ &\doteq 0.992 \text{ or } 99.2\% \quad \longleftarrow \text{greater than 99\%}\end{aligned}$$

The coin has to be tossed 7 times for the probability of obtaining at least one head to be greater than or equal to 99%.

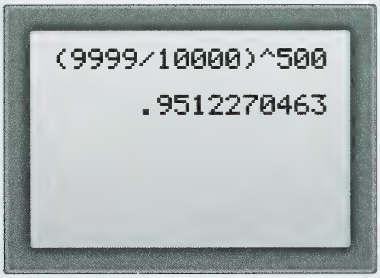
9. **a.** $P(\text{grand prize}) = \dfrac{1}{10\,000}$

b. First, find the probability of not winning on any draw.

$$P(\text{not winning}) = \frac{9999}{10\,000}$$

Now, find the probability of not winning any of the 500 prizes.

$$\begin{aligned}P(\text{not winning any prize}) &= \left(\frac{9999}{10\,000}\right)^{500}\\ &\doteq 0.951 \text{ or } 95.1\%\end{aligned}$$



```
(9999/10000)^500
      .9512270463
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The complement of not winning any prize is winning at least one prize.

$$\begin{aligned}\therefore\ P(\text{winning at least 1 prize}) &= 1 - P(\text{not winning any prize})\\ &\doteq 1-0.951\\ &\doteq 0.049 \text{ or } 4.9\%\end{aligned}$$

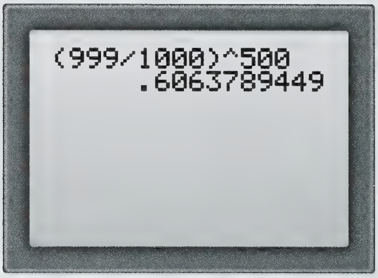
The probability of winning at least one prize is about 4.9%.

c.
$$\begin{aligned}P(\text{grand prize}) &= \frac{10}{10\,000}\\ &= \frac{1}{1000}\end{aligned}$$

d. First, determine your chances of not winning the prize on a given draw.

$$\begin{aligned} P(\text{not winning}) &= 1 - \frac{1}{1000} \\ &= \frac{999}{1000} \end{aligned}$$

Now, determine the probability of not winning a single prize out of 500 draws.



```
(999/1000)^500
    .6063789449
```

$$\begin{aligned} P(\text{not winning any prize}) &= \left(\frac{999}{1000}\right)^{500} \\ &\doteq 0.606 \text{ or } 60.6\% \end{aligned}$$

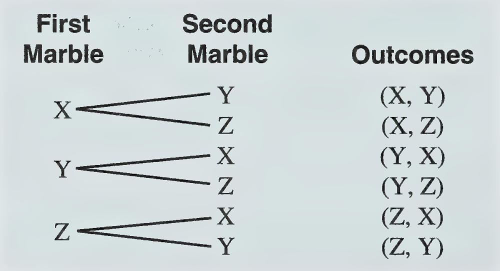
The complement of not winning any prizes is winning at least one prize.

$$\begin{aligned} \therefore\ P(\text{winning at least 1 prize}) &= 1 - P(\text{not winning any prize}) \\ &\doteq 1 - 0.606 \\ &\doteq 0.394 \text{ or } 39.4\% \end{aligned}$$

The probability of winning at least one prize is about 39.4%.

b. Textbook exercises 10 and 11 of "Exercises: Extending Your Thinking," p. 33

10. a. Draw a tree diagram to display the sample space.



There are 4 outcomes in A: (X, Y), (X, Z), (Y, X), and (Z, X).

$$\begin{aligned} \therefore\ P(A) &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

b. If marble *X* has been selected, there are only two marbles left in the bag, one of which is *Y*.

$$\therefore P(B|A) = \frac{1}{2}$$

c. *A* and *B* consists of the 2 outcomes: (X, Y) and (Y, X).

$$\begin{aligned}\therefore P(A \text{ and } B) &= \frac{2}{6} \\ &= \frac{1}{3}\end{aligned}$$

d. Since $\frac{1}{3} = \frac{2}{3} \times \frac{1}{2}$, $P(A \text{ and } B) = P(A) \times P(B|A)$.

11. a. Let *A* and *B* be the following events:

A: "The first card is a diamond."
B: "The second card is a diamond."

There are 13 diamonds in a standard deck.

$$\begin{aligned}P(A) &= \frac{13}{52} \\ &= \frac{1}{4}\end{aligned} \qquad \begin{aligned}P(B|A) &= \frac{12}{51} \\ &= \frac{4}{17}\end{aligned}$$

If the first card is a diamond, there are only 12 diamonds left and 51 cards in the deck.

$$\begin{aligned}\therefore P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{1}{4} \times \frac{4}{17} \\ &= \frac{1}{17}\end{aligned}$$

The probability of drawing two diamonds is $\frac{1}{17}$.

Activity 4 (continued)

b. Let A and B be the following events:

A: "A red card is drawn first."
B: "A red card is drawn second."

There are 26 red cards (hearts and diamonds) in a standard deck.

$$\begin{aligned} P(A) &= \frac{26}{52} \\ &= \frac{1}{2} \end{aligned} \qquad P(B|A) = \frac{25}{51}$$

If the first card is red, there is one fewer red card and one fewer card in the deck for the second draw.

$$\begin{aligned} \therefore\ P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{1}{2} \times \frac{25}{51} \\ &= \frac{25}{102} \end{aligned}$$

The probability of drawing two red cards is $\frac{25}{102}$.

c. Let A and B be the following events:

A: "The first card is a face card."
B: "The second card is a face card."

There are 12 face cards (jacks, queens, and kings) in a standard deck.

$$\begin{aligned} P(A) &= \frac{12}{52} \\ &= \frac{3}{13} \end{aligned} \qquad P(B|A) = \frac{11}{51}$$

$$\begin{aligned} \therefore\ P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{\overset{1}{\cancel{3}}}{13} \times \frac{11}{\underset{17}{\cancel{51}}} \\ &= \frac{11}{221} \end{aligned}$$

The probability of drawing two face cards is $\frac{11}{221}$.

d. Let *A* and *B* be the following events:

A: "The first card is a 5."
B: "The second card is a 5."

There are four 5s in a standard deck.

$$P(A) = \frac{4}{52} = \frac{1}{13} \qquad P(B|A) = \frac{3}{51} = \frac{1}{17}$$

$$\begin{aligned}\therefore\ P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{1}{13} \times \frac{1}{17} \\ &= \frac{1}{221}\end{aligned}$$

The probability of drawing two 5s is $\frac{1}{221}$.

e. Let *A* and *B* be the following events:

A: "The first card is not a club."
B: "The second card is not a club."

There are 39 cards that are not clubs in a standard deck.

$$P(A) = \frac{39}{52} = \frac{3}{4} \qquad P(B|A) = \frac{38}{51}$$

$$\begin{aligned}\therefore\ P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{19}{\cancel{38}}}{\underset{17}{\cancel{51}}} \\ &= \frac{19}{34}\end{aligned}$$

The probability that neither card is a club is $\frac{19}{34}$.

5. Textbook exercise "Communicating the Ideas," p. 33

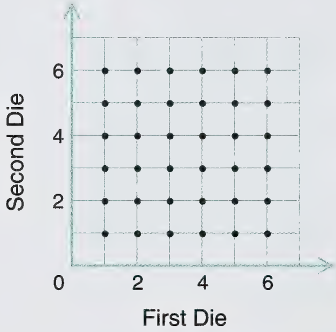
Two events are independent if the occurrence of one event does not affect the occurrence of the other. For example, tossing a coin twice are independent events. Whether the coin turns up heads or tails on the first toss does not affect the outcome of the second toss.

Two events are dependent if the occurrence of one event does affect the occurrence of a second event. For example, you have a bag containing one red marble and two white marbles. If two marbles are drawn at random from the bag, one at a time, and the first marble is not replaced before the second is drawn, the probability of a white marble on the second draw is 100% if the first marble is red. If the first marble drawn is white, there are one red and one white marble left in the bag. Therefore, the probability of drawing a white marble on the second draw is only 50%.

Activity 5: Mutually Exclusive Events

1. Textbook exercises 1 to 5 of "Investigation: 'OR' Probabilities," p. 34

1. Use a grid to list the outcomes of the sample space.



2. List the outcomes in events X, Y, and Z.

X: $(1,4),(2,3),(3,2),(4,1)$

Y: $(2,6),(3,5),(4,4),(5,3),(6,2)$

Z: $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$

a. $P(\text{X})=\frac{4}{36}$

$=\frac{1}{9}$

b. $P(\text{Y})=\frac{5}{36}$

c. $P(\text{X and Y}) = 0$

d. $P(\text{X or Y}) = \dfrac{9}{36}$

$= \dfrac{1}{4}$

e. $P(\text{X or Y}) = P(\text{X}) + P(\text{Y})$

In this case, there is no relation between $P(\text{X or Y})$ and $P(\text{X and Y})$.

3. **a.** $P(\text{X}) = \dfrac{4}{36}$

$= \dfrac{1}{9}$

b. $P(\text{Z}) = \dfrac{6}{36}$

$= \dfrac{1}{6}$

c. $P(\text{X and Z}) = 0$

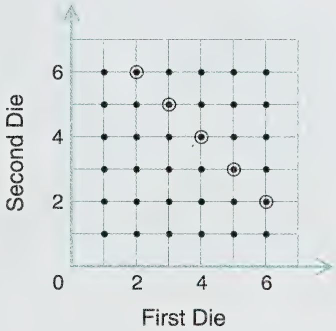
d. $P(\text{X or Z}) = \dfrac{10}{36}$

$= \dfrac{5}{18}$

e. $P(\text{X or Z}) = P(\text{X}) + P(\text{Z})$

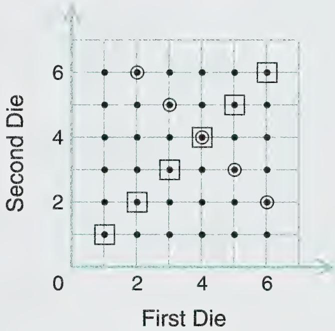
In this case, there is no relation between $P(\text{X or Z})$ and $P(\text{X and Z})$.

4. **a.**



$P(\text{Y}) = \dfrac{5}{36}$

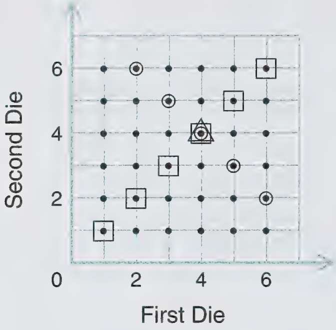
b.



$P(\text{Z}) = \dfrac{6}{36}$

$= \dfrac{1}{6}$

c.



$P(\text{Y and Z}) = \frac{1}{36}$

d. $P(\text{Y or Z}) = \frac{10}{36}$

$= \frac{5}{18}$

e. Event Y and event Z share a common outcome, $(4, 4)$. Because $(4, 4)$ is counted in $P(\text{Y})$ and in $P(\text{Z})$, you end up counting this outcome twice when you add $P(\text{Y})$ and $P(\text{Z})$. So, to remove the extra outcome, simply subtract $P(\text{Y and Z})$ from the sum of $P(\text{Y})$ and $P(\text{Z})$ to determine $P(\text{Y or Z})$.

$$P(\text{Y or Z}) = P(\text{Y}) + P(\text{Z}) - P(\text{Y and Z})$$

5. In exercise 4, the events have a common outcome. There are no common outcomes for the events in exercises 2 and 3.

2. Textbook exercises 1 and 2 of “Exercises: Checking Your Skills,” pp. 37 and 38

1. **a.** The events are mutually exclusive. All doubles give an even sum, whereas 11 is an odd number.
   b. The events are not mutually exlusive. The seven of clubs is a common outcome.
   c. The events are not mutually exclusive. The common outcomes are 91, 93, 95, 97, and 99.
   d. The events are mutually exclusive.
   e. The events are mutually exclusive. Diamonds are red cards.

2. **a.** $P(A \text{ or } \bar{A}) = 100\%$ since A or $\bar{A}$ is the entire sample space.

   b. $P(A \text{ and } \bar{A}) = 0\%$ since A or $\bar{A}$ have no common outcomes. Either A occurs or A does not occur. Event A cannot occur and not occur at the same time.

3. Textbook exercises 3 to 8 of "Exercises: Checking Your Skills," pp. 38 and 39

3. a. Let A and B represent the following events:

A: "The number is greater than 7."
B: "The number is less than 4."

List the outcomes in each event.

A: 8, 9, 10
B: 1, 2, 3
A and B: None

Events A and B are mutually exclusive.

$$\begin{aligned}\therefore P(A \text{ or } B) &= P(A)+P(B)\\ &= \frac{3}{10}+\frac{3}{10}\\ &= \frac{6}{10}\\ &= \frac{3}{5}\end{aligned}$$

The probability that the number is greater than 7 or less than 4 is $\frac{3}{5}$.

b. Let A and B represent the following events:

A: "The number is odd."
B: "The number is a perfect square."

List the outcomes in each event.

A: 1, 3, 5, 7, 9
B: 1, 4, 9
A and B: 1, 9

The events are not mutually exclusive.

$$\begin{aligned}\therefore P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B)\\ &= \frac{5}{10}+\frac{3}{10}-\frac{2}{10}\\ &= \frac{6}{10}\\ &= \frac{3}{5}\end{aligned}$$

The probability that the number is odd or a perfect square is $\frac{3}{5}$.

c. Let A and B represent the following events:

A: "The number is odd."
B: "The number is a multiple of 5."

List the outcomes in each event.

A: 1, 3, 5, 7, 9
B: 5, 10
A and B: 5

The events are not mutually exclusive.

$$\begin{aligned} \therefore\ P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B) \\ &= \frac{5}{10}+\frac{2}{10}-\frac{1}{10} \\ &= \frac{6}{10} \\ &= \frac{3}{5} \end{aligned}$$

The probability that the number is odd or a multiple of 5 is $\frac{3}{5}$.

d. Let A and B represent the following events:

A: "The number is even."
B: "The number is a multiple of 3."

List the outcomes in each event.

A: 2, 4, 6, 8, 10
B: 3, 6, 9
A and B: 6

The events are not mutually exclusive.

$$\begin{aligned} \therefore\ P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B) \\ &= \frac{5}{10}+\frac{3}{10}-\frac{1}{10} \\ &= \frac{7}{10} \end{aligned}$$

The probability that the number is even or a multiple of 3 is $\frac{7}{10}$.

e. Let *A* and *B* represent the following events:

A: “The number is odd.”
B: “The number is greater than 5.”

List the outcomes in each event.

A: 1, 3, 5, 7, 9
B: 6, 7, 8, 9, 10
A and *B*: 7, 9

The events are not mutually exclusive.

$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B)\\ &= \frac{5}{10}+\frac{5}{10}-\frac{2}{10}\\ &= \frac{8}{10}\\ &= \frac{4}{5}\end{aligned}$$

The probability that the number is odd or greater than 5 is $\frac{4}{5}$.

4. a. Let *A* and *B* represent the following events:

A: “The card is a heart.”
B: “The card is a spade.”

The events are mutually exclusive.

$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A)+P(B)\\ &= \frac{13}{52}+\frac{13}{52}\\ &= \frac{26}{52}\\ &= \frac{1}{2}\end{aligned}$$

There are 13 hearts and 13 spades in a standard deck of cards.

The probability that the card is a heart or spade is $\frac{1}{2}$.

Activity 5 (continued)

b. Let A and B represent the following events:

A: "The card is an ace."
B: "The card is a face card."

The events are mutually exclusive.

$$\begin{aligned}\therefore P(A \text{ or } B) &= P(A) + P(B)\\ &= \frac{4}{52} + \frac{12}{52}\\ &= \frac{16}{52}\\ &= \frac{4}{13}\end{aligned}$$

There are 4 aces and 12 face cards in a standard deck of cards.

The probability that the card is an ace or a face card is $\frac{4}{13}$.

c. Let A and B represent the following events:

A: "The card is a heart."
B: "The card is a face card."

The events are not mutually exclusive. There is a jack, queen, and king of hearts.

$$\begin{aligned}\therefore P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B)\\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}\\ &= \frac{22}{52}\\ &= \frac{11}{26}\end{aligned}$$

The probability that the card is a heart or a face card is $\frac{11}{26}$.

d. Let A and B represent the following events:

A: "The card is an ace."
B: "The card is a spade."

The events are not mutually exclusive. There is an ace of spades.

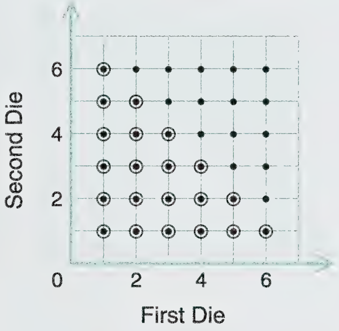
$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B)\\ &= \frac{4}{52}+\frac{13}{52}-\frac{1}{52}\\ &= \frac{16}{52}\\ &= \frac{4}{13}\end{aligned}$$

The probability that the card is an ace or a spade is $\frac{4}{13}$.

5. **a.** Let A represent the following event:

A: "The sum of the dice is less than or equal to 7."

Use a grid to represent the sample space, circling the favourable outcomes.



There are 21 outcomes in A.

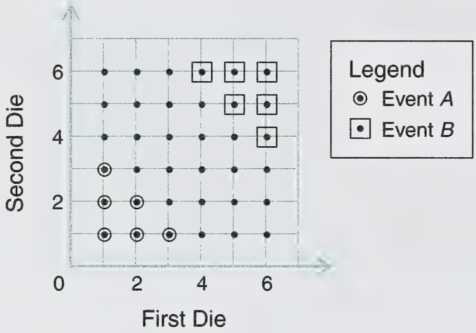
$$\begin{aligned}\therefore\ P(A) &= \frac{21}{36}\\ &= \frac{7}{12}\end{aligned}$$

The probability that the sum is less than or equal to 7 is $\frac{7}{12}$.

b. Let A and B represent the following events:

A: "The sum of the dice is less than 5."
B: "The sum of the dice is greater than 9."

Use a grid to represent the sample space.



Events A and B are mutually exclusive.

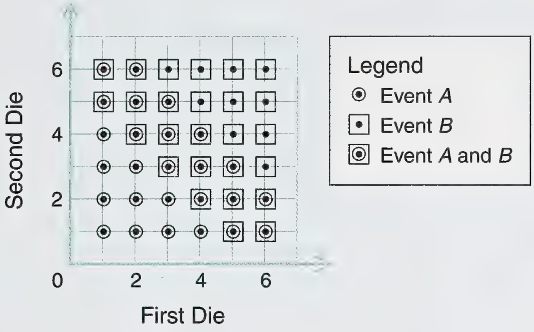
$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A)+P(B)\\ &= \frac{6}{36}+\frac{6}{36}\\ &= \frac{12}{36}\\ &= \frac{1}{3}\end{aligned}$$

The probability that the sum of the dice is less than 5 or greater than 9 is $\frac{1}{3}$.

c. Let A and B represent the following events:

A: "The sum of the dice is less than 9."
B: "The sum of the dice is greater than 5."

Use a grid to represent the sample space.



Events A and B are not mutually exclusive. There are 16 common outcomes.

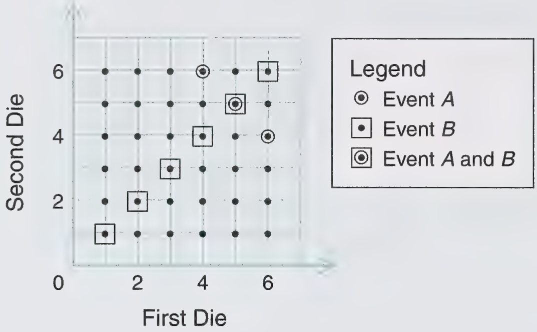
$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B)\\ &= \frac{26}{36} + \frac{26}{36} - \frac{16}{36}\\ &= \frac{36}{36}\\ &= 1\end{aligned}$$

The probability that the sum of the dice is less than 9 or greater than 5 is 1, or 100%.

d. Let A and B represent the following events:

A: "The sum of the dice is 10."
B: "Doubles are rolled."

Use a grid to represent the sample space.



Events A and B are not mutually exclusive. There is 1 common outcome.

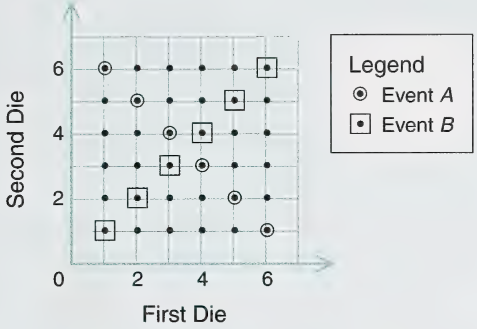
$$\begin{aligned}\therefore\ P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B)\\ &= \frac{3}{36} + \frac{6}{36} - \frac{1}{36}\\ &= \frac{8}{36}\\ &= \frac{2}{9}\end{aligned}$$

The probability that the sum of the dice is 10 or doubles are rolled is $\frac{2}{9}$.

e. Let A and B represent the following events:

A: "The sum of the dice is 7."
B: "Doubles are rolled."

Use a grid to represent the sample space.



Events A and B are mutually exclusive.

$$\therefore P(A \text{ or } B) = P(A) + P(B)$$
$$= \frac{6}{36} + \frac{6}{36}$$
$$= \frac{12}{36}$$
$$= \frac{1}{3}$$

The probability that the sum of the dice is 7 or doubles are rolled is $\frac{1}{3}$.

6. Let A and B represent the following events:

A: "The customer learned about the sale from the newspaper."
B: "The customer learned about the sale from TV."
A and B: "The customer learned about the sale from both sources."

a. $P(A \text{ and } B) = \frac{30}{200}$

$$= \frac{3}{20} \text{ or } 15\%$$

The probability that a randomly selected customer saw the advertisement in the newspaper and on TV is 15%.

b. $$\begin{aligned} P(A \text{ or } B) &= P(A)+P(B)-P(A \text{ and } B) \\ &= \frac{60}{200}+\frac{50}{200}-\frac{30}{200} \\ &= \frac{80}{200} \\ &= \frac{2}{5} \text{ or } 40\% \end{aligned}$$

The probability that a randomly selected customer saw the advertisement in at least one of the media is 40%.

c. There were 50 people who saw the sale advertised on TV, 30 of which also saw the sale advertised in the newspaper. So, only 20 people saw the advertisement solely on TV.

$$\begin{aligned} \therefore P(\text{on TV, not in newspaper}) &= \frac{20}{200} \\ &= \frac{1}{10} \text{ or } 10\% \end{aligned}$$

7. Let *A* and *B* represent the following events:

A: “Team A wins a game.”
B: “Team B wins a game.”

$$\begin{aligned} \therefore P(A)=0.55 \text{ and } P(B) &= 1-P(A) \\ &= 1-0.55 \\ &= 0.45 \end{aligned}$$

The probability that team A sweeps the series is

$$\begin{aligned} P(\text{A sweeps}) &= 0.55\times0.55\times0.55\times0.55 \\ &= (0.55)^4 \\ &\doteq 0.0915 \end{aligned}$$

The probability that team B sweeps the series is

$$\begin{aligned} P(\text{B sweeps}) &= (0.45)^4 \\ &\doteq 0.0410 \end{aligned}$$

Because both teams cannot end up sweeping the series,

$$\begin{aligned} P(\text{A sweeps or B sweeps}) &= P(\text{A sweeps}) + P(\text{B sweeps}) \\ &\doteq 0.0915 + 0.0410 \\ &\doteq 0.1325 \text{ or } 13.25\% \end{aligned}$$

The probability that either team wins the series in the first four games is about 13.25%.

8. Let A and B represent the following events:

A: "A white marble is drawn."
B: "A green marble is drawn."

Because the marbles are replaced, the probabilities for either draw are

$$P(A) = \frac{3}{8} \text{ and } P(B) = \frac{5}{8}$$

$$\therefore\ \begin{aligned} P(\text{2 whites}) &= P(A) \times P(A) \\ &= \frac{3}{8} \times \frac{3}{8} \\ &= \frac{9}{64} \end{aligned} \quad \text{and} \quad \begin{aligned} P(\text{2 greens}) &= P(B) \times P(B) \\ &= \frac{5}{8} \times \frac{5}{8} \\ &= \frac{25}{64} \end{aligned}$$

$$\therefore\ \begin{aligned} P(\text{2 whites or 2 greens}) &= P(\text{2 whites}) + P(\text{2 greens}) \\ &= \frac{9}{64} + \frac{25}{64} \\ &= \frac{34}{64} \\ &= \frac{17}{32} \end{aligned}$$

The probability that both marbles are white or both are green is $\frac{17}{32}$.

4. Textbook exercise "Communicating the Ideas," p. 39

Answers will vary. A sample answer is given.

Events A and B are mutually exclusive if they cannot occur at the same time. If A and B are mutually exclusive, $P(\text{A and B}) = 0$ and $P(\text{A or B}) = P(\text{A}) + P(\text{B})$.

If events A and B are not mutually exclusive, they can occur at the same time. If this is the case, $P(\text{A and B}) \neq 0$ and $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$.

Module Review

1. Textbook exercises 1.a. to 1.f. of Part A of "What Should I Be Able to Do?" p. 43

1. a. **First Spin**

		Win, W	Win, W	Lose, L
Second Spin	Win, W	(W, W)	(W, W)	(L, W)
	Win, W	(W, W)	(W, W)	(L, W)
	Lose, L	(W, L)	(W, L)	(L, L)

b. i. $P\left(2\,\mathrm{W}\right)=\frac{4}{9}$

ii. $P\left(\text{at least }1\,\mathrm{W}\right)=\frac{8}{9}$

c. i. $$\begin{aligned}P\left(\mathrm{A}\right)&=\frac{6}{9}\\&=\frac{2}{3}\end{aligned}$$

ii. $$\begin{aligned}P\left(\mathrm{B}\right)&=\frac{6}{9}\\&=\frac{2}{3}\end{aligned}$$

iii. $$\begin{aligned}P\left(\text{A and B}\right)&=P\left(\mathrm{A}\right)\times P\left(\mathrm{B}\right)\\&=\frac{2}{3}\times\frac{2}{3}\\&=\frac{4}{9}\end{aligned}$$

iv. $$\begin{aligned}P\left(\text{A or B}\right)&=P\left(\mathrm{A}\right)+P\left(\mathrm{B}\right)-P\left(\text{A and B}\right)\\&=\frac{2}{3}+\frac{2}{3}-\frac{4}{9}\\&=\frac{6}{9}+\frac{6}{9}-\frac{4}{9}\\&=\frac{8}{9}\end{aligned}$$

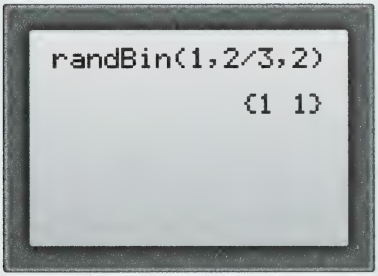
d. The results for parts b and c are the same.

e. Answers will vary. A sample simulation is given.

Enter the data for the first experiment, and record the results.

Select the PRB menu.

MATH ◀ 7 (7:randBin() 1 , 2 ÷ 3 , 2) ENTER

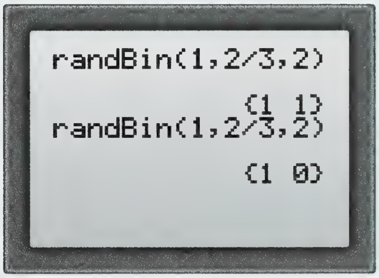


```
randBin(1,2/3,2)
               {1 1}
```

The ordered pair $(1,1)$ represents two wins.

Enter the data for the second experiment, and record the results. **Note:** Because the last entry is saved in the graphing calculator's memory, simply press the following to carry out the second experiment. Then repeat this method 13 times for a total of 15 trials.

2nd [ENTRY] ENTER



```
randBin(1,2/3,2)
               {1 1}
randBin(1,2/3,2)
               {1 0}
```

The ordered pair $(1,0)$ represents a win on the first toss and a loss on the second toss.

The results of repeating this experiement 15 times is as follows.

Outcome	Tally	Number
$(1,1)$	𝍸 ||	7
$(1,0)$	||	2
$(0,1)$	|||	3
$(0,0)$	|||	3

i. $P(2\,\mathrm{W})=\frac{7}{15}$ or about 46.7%

The experimental probability of the spinner landing on "win" on the first and second spin is $\frac{7}{15}$, or about 46.7%.

ii. $P(\text{at least } 1\,\text{W}) = \frac{12}{15}$

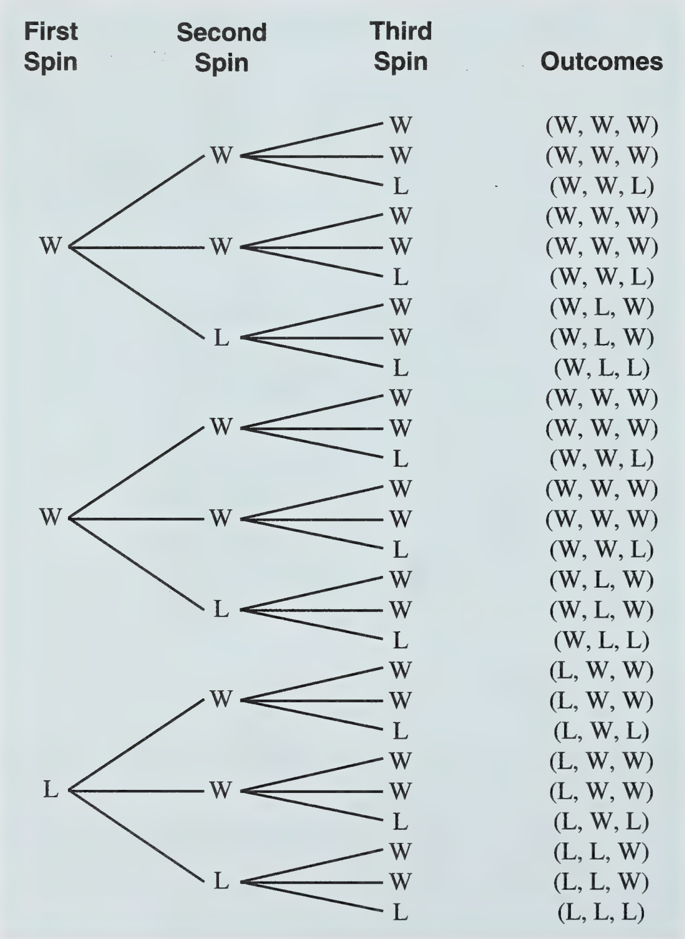
$= \frac{4}{5}$ or 80%

The experimental probability of the spinner landing on "win" on the first or second spin is $\frac{4}{5}$, or 80%.

f. The experimental probabilities are close to the theoretical probabilities. The experimental probabilities should approach the theoretical probabilities as more trials are conducted.

2. Textbook exercises 2 to 8 of Part B of "What Should I Be Able to Do?" pp. 44 and 45

2. a.



b. $P(\mathrm{A}) = \frac{8}{27}$

c. The complement of *A* is the spinner lands on "lose" at least once.

d.
$$
\begin{aligned}
P(\bar{A}) &= 1 - P(A) \\
&= 1 - \frac{8}{27} \\
&= \frac{27}{27} - \frac{8}{27} \\
&= \frac{19}{27}
\end{aligned}
$$

3. a. There are no restrictions.

26	10	26	10	26	10
letter	digit	letter	digit	letter	digit

Use the Fundamental Counting Principle.

$$
\begin{aligned}
\text{Total number} &= 26 \times 10 \times 26 \times 10 \times 26 \times 10 \\
&= 26^3 \times 10^3 \\
&= 17\,576\,000
\end{aligned}
$$

There are 17 576 000 different postal codes.

b. If the first letter must be S, there is only 1 choice for the first letter.

1	10	26	10	26	10
letter	digit	letter	digit	letter	digit

$$
\begin{aligned}
\therefore\ \text{Total number} &= 1 \times 10 \times 26 \times 10 \times 26 \times 10 \\
&= 676\,000
\end{aligned}
$$

There are 676 000 different postal codes where the first letter is S.

c. If the first letter must be S and the first digit must be a 7, there is only 1 choice for each of the first two positions.

1	1	26	10	26	10
letter	digit	letter	digit	letter	digit

$$
\begin{aligned}
\therefore\ \text{Total number} &= 1 \times 1 \times 26 \times 10 \times 26 \times 10 \\
&= 67\,600
\end{aligned}
$$

There are 67 600 different postal codes where the first letter is S and the first digit is 7.

4. **a.** Assuming the digits can be repeated, there are $9 \times 9 \times 9 \times 9 = 6561$ different four-digit numbers possible.

b. If no digit can be repeated, there is one fewer choice for each placeholder. Therefore, there are $9 \times 8 \times 7 \times 6 = 3024$ different four-digit numbers possible.

c. Let *A* be the event that no digit is repeated. Therefore, the complement, $\overline{A}$, is that at least 1 digit is repeated.

$$\begin{aligned} P\left(\overline{A}\right) &= 1 - P(A) \\ &= 1 - \frac{3024}{6561} \\ &= \frac{6561}{6561} - \frac{3024}{6561} \\ &= \frac{3537}{6561} \\ &= \frac{131}{243} \text{ or about } 53.9\% \end{aligned}$$

The probability that at least one digit is repeated is about 53.9%.

5. Because the cards are replaced, the events are independent.

a. On each draw, the probability of a club is $\frac{13}{52}$ or $\frac{1}{4}$.

$$\begin{aligned} \therefore\ P(3 \text{ clubs}) &= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{64} \end{aligned}$$

b. On each draw, the probability of a red card is $\frac{26}{52}$ or $\frac{1}{2}$.

$$\begin{aligned} \therefore\ P(3 \text{ red cards}) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

c. There are 12 face cards (jacks, queens, and kings) in a standard deck. On each draw, the probability of a face card is $\frac{12}{52}$ or $\frac{3}{13}$.

$$\begin{aligned}\therefore\ P(3\text{ face cards}) &= \frac{3}{13}\times\frac{3}{13}\times\frac{3}{13}\\ &= \frac{27}{2197}\end{aligned}$$

d. First, determine the probability that no card is a heart. There are 39 cards in the other suits. Therefore, on each draw, the probability of a card that is not a heart is $\frac{39}{52}$ or $\frac{3}{4}$.

$$\begin{aligned}\therefore\ P(\text{no hearts}) &= \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\\ &= \frac{27}{64}\end{aligned}$$

The complement of "no hearts" is "at least one heart."

$$\begin{aligned}\therefore\ P(\text{at least one heart}) &= 1-P(\text{no hearts})\\ &= 1-\frac{27}{64}\\ &= \frac{64}{66}-\frac{27}{64}\\ &= \frac{37}{64}\end{aligned}$$

6. Let A and B represent the following events:

A: "The student takes vitamin C daily."
B: "The student catches a cold."

You are told that $P(A)=20\%$, $P(B)=60\%$, and $P(A\text{ and }B)=8\%$.

If A and B are independent events, $P(A\text{ and }B)=P(A)\times P(B)$. Check.

LS	RS
$P(A\text{ and }B)$	$P(A)\times P(B)$
$=8\%$	$=20\%\times 60\%$
$=0.08$	$=0.02\times 0.60$
	$=0.12$

LS $\neq$ RS

Since $P(A\text{ and }B)\neq P(A)\times P(B)$, taking vitamin C and catching a cold are not independent events.

7. **a.** If the 12 face cards are removed from the deck, there are $52-12=40$ cards remaining, 10 in each suit.

Let A and B represent the following events:

A: "The card is a spade."
B: "The card is red."

$$\therefore\ P(A)=\frac{10}{40} \text{ and } P(B)=\frac{20}{40}$$

Events A and B are mutually exclusive.

$$\begin{aligned}P(A \text{ or } B)&=P(A)+P(B)\\&=\frac{10}{40}+\frac{20}{40}\\&=\frac{30}{40}\\&=\frac{3}{4}\end{aligned}$$

The probability of a spade or a red card is $\frac{3}{4}$.

b. Let A and B represent the following events:

A: "The card is a 4."
B: "The card is a club."

$$\therefore\ P(A)=\frac{4}{40} \text{ and } P(B)=\frac{10}{40}$$

Events A and B are not mutually exclusive. The 4 of clubs is a common outcome.

$$\begin{aligned}\therefore\ P(A \text{ or } B)&=P(A)+P(B)-P(A \text{ and } B)\\&=\frac{4}{40}+\frac{10}{40}-\frac{1}{40}\\&=\frac{13}{40}\end{aligned}$$

The probability of drawing a 4 or a club is $\frac{13}{40}$.

c. Let A and B represent the following events:

A: "Drawing a diamond."
B: "Drawing a red 8 (hearts or diamonds)"

$$\therefore\ P(A)=\frac{10}{40}\text{ and }P(B)=\frac{2}{40}$$

Events A and B are not mutually exclusive. The 8 of diamonds is a common outcome.

$$\begin{aligned}\therefore\ P(A\text{ or }B)&=P(A)+P(B)-P(A\text{ and }B)\\&=\frac{10}{40}+\frac{2}{40}-\frac{1}{40}\\&=\frac{11}{40}\end{aligned}$$

The probability of drawing a diamond or a red 8 is $\frac{11}{40}$.

8. $P(\text{José wins})=0.75$

$$\begin{aligned}P(\text{José loses})&=1-0.75\\&=0.25\end{aligned}$$

$$\begin{aligned}P(\text{José wins, then loses})&=0.75\times0.25\\&=0.1875\end{aligned}$$

$$\begin{aligned}P(\text{José loses, then wins})&=0.25\times0.75\\&=0.1875\end{aligned}$$

$$\begin{aligned}\therefore\ P(\text{Jose}'\text{ wins the first or second match, but not both})&=0.1875+0.1875\\&=0.3750\end{aligned}$$

The probability José wins exactly one game is 0.375.

Module Review (continued)

Enrichment

1. Let F_1, F_2, and F_3 represent the following events:

F_1: “The first card is a face card.”

F_2: “The second card is a face card.”

F_3: “The third card is a face card.”

$$\begin{aligned} P(\text{drawing 3 face cards}) &= P(F_1)\times P(F_2|F_1)\times P(F_3|F_2|F_1) \\ &= \frac{12}{52}\times\frac{11}{51}\times\frac{10}{50} \\ &= \frac{\overset{1}{\cancel{3}}}{13}\times\frac{11}{\underset{17}{\cancel{51}}}\times\frac{1}{5} \\ &= \frac{11}{1105} \text{ or about } 1.0\% \end{aligned}$$

The probability of drawing three face cards is $\frac{11}{1105}$, or about 1.0%.

2. Let K_1, Q_2, and Q_3 represent the following events:

K_1: “The first card is a king.”

Q_2: “The second card is a queen.”

Q_3: “The third card is a queen.”

$$\begin{aligned} P(\text{king followed by 2 queens}) &= P(K_1)\times P(Q_2|K_1)\times P(Q_3|K_1|Q_2) \\ &= \frac{4}{52}\times\frac{4}{51}\times\frac{3}{50} \\ &= \frac{1}{13}\times\frac{\overset{2}{\cancel{4}}}{\underset{17}{\cancel{51}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{25}{\cancel{50}}} \\ &= \frac{2}{5525} \end{aligned}$$

However, the king could be the second card or the third card. Each of these events are equally probable.

$$\begin{aligned}\therefore\ P(\text{king and 2 queens}) &= 3 \times \frac{2}{5525}\\ &= \frac{6}{5525} \text{ or about } 0.1\%\end{aligned}$$

The probability of drawing a king and two queens is $\frac{6}{5525}$, or about 0.1%.

3. Let R_1 and W_2 represent the following events:

R_1: "A red marble is selected first."

W_2: "A white marble is selected second."

$$\begin{aligned}P(\text{R}_1 \text{ and } \text{W}_2) &= P(\text{R}_1) \times P(\text{W}_2 | \text{R}_1)\\ &= \frac{3}{5} \times \frac{2}{4}\\ &= \frac{3}{5} \times \frac{1}{2}\\ &= \frac{3}{10}\end{aligned}$$

You could draw a white marble first and a red marble second. The probability of this event is the same as drawing a red first and a white second.

$$\begin{aligned}\therefore\ P(\text{red and white}) &= \overset{1}{\cancel{2}} \times \frac{3}{\underset{5}{\cancel{10}}}\\ &= \frac{3}{5} \text{ or } 60\%\end{aligned}$$

The probability of drawing one red and one white marble is $\frac{3}{5}$, or 60%.

Completing the Project

1. **Textbook exercise 9 (first bullet) of Part C of "What Should I Be Able to Do?," p. 46**

9. Answers will vary. A sample answer is given.

The student has made an error in the calculation of the number of passwords. It was in assuming that passwords like 282 or 118 are to be excluded. The requirement is that passwords consisting of all the same digit are to be excluded. That is, only exclude passwords like 111, 3333, 5555.

To count 3-digit passwords, use the Fundamental Counting Principle to find the total number of possible passwords. This gives $10 \times 10 \times 10 = 1000$ possible three-digit passwords. From this, you need to subtract the passwords, like 000, 111, …, 999, that are to be excluded. Since there are 10 passwords of this type, there will be 990 available three-digit passwords.

The number of four- and five-digit passwords should be calculated in the same way.

This error affects the remaining calculations for Proposal A. It may also affect other parts of the student's analysis of Proposal A.

2. **Textbook exercise 10 of Part C of "What Should I Be Able to Do?," pp. 47 and 48**

10. The reports that students are to write are detailed on page 41 of the textbook. There are six points that need to be addressed. These points are shown in the following tables along with an analysis of the sample reports on page 47 of the textbook. Your analysis of the sample work should have indicated the items found in the table and should be reported in paragraph form. Notice that since the reports (and not the answers to exercises 1 to 6) are to be submitted, the work carried out in exercises 1 to 6 needs to be included (where appropriate) in the report.

Proposal A

Point	[illegible]
The number of possible passwords available	The number of available passwords is listed, but it is incorrect.
The probability of users forgetting their passwords	The probability of forgetting a password is given, but it is incorrect. The given probability is actually an incorrect calculation of the probability of guessing the password.
The level of security offered	The level of security is mentioned, but it was not calculated nor quantified numerically.
Other considerations	A shortage of passwords based on a unique password per user was noted.
Suggestions for improvement	A suggestion that longer passwords be included was noted.
All relevant calculations	Calculations are not shown in the report.

Here are some things that could have been done to improve the report:

- **The Number of Possible Passwords**

  Calculate and list the correct number of available passwords as 110 970.

- **The Probability of Users Forgetting Their Passwords**

  Calculate the probability of users forgetting their password using the data from the table on page 40 of the textbook. You may wish to report this as calculated in question 1 for the probability that one of the 10 000 users will forget a password on one day.

  The probability of remembering a three-digit password is $1-0.0001=0.9999$. So, the probability of 10 000 people remembering their three-digit password is $(0.9999)^{10\,000} \doteq 0.3679$ or 36.79%. Thus, there is a 63.21% chance of at least one person forgetting his or her three-digit password each day.

  The probability of remembering a four-digit password is $1-0.000\,15=0.999\,85$. So, the probability of 10 000 people remembering their four-digit password is $(0.999\,85)^{10\,000} \doteq 0.2231$ or 22.31%. Thus, there is a 77.69% chance of at least one person forgetting his or her four-digit password each day.

The probability of remembering a five-digit password is $1-0.0002=0.9998$. So, the probability of 10 000 people remembering their five-digit password is $(0.9998)^{10\,000} \doteq 0.1353$ or 13.53%. Thus, there is a 86.47% chance of at least one person forgetting his or her five-digit password each day.

$$\begin{aligned} P(3\text{-digit password}) &= \frac{990}{110\,970} \\ &\doteq 0.0089 \end{aligned}$$

$$\begin{aligned} P(4\text{-digit password}) &= \frac{9990}{110\,970} \\ &\doteq 0.0900 \end{aligned}$$

$$\begin{aligned} P(5\text{-digit password}) &= \frac{99\,990}{110\,970} \\ &\doteq 0.9011 \end{aligned}$$

The probability of someone forgetting a three-, four-, or five-digit password is

$$\begin{aligned} P &\doteq 0.089(0.6321)+0.0900(0.7769)+0.9011(0.8647) \\ &\doteq 0.0056+0.0070+0.7792 \\ &\doteq 0.7918 \text{ or } 79.18\% \end{aligned}$$

- **The Level of Security Offered**

  Calculate and list the probability of a password being guessed. Note that a smaller probability of guessing a password means a more secure system.

- **Other Considerations**

  List any other considerations you may have.

- **Suggestions for Improvement**

  Calculate and list the improved security for longer passwords and the increased risk of forgetting the password. List any other improvements you may have.

- **All Relevant Calculations**

  Show the calculations.

Proposal B

Point	[illegible]
The number of possible passwords available	The correct number of available passwords is listed.
The probability of users forgetting their passwords	The probability of forgetting a password is mentioned as being lower, but no value is given.
The level of security offered	The level of security is mentioned, but it is not calculated. It is labelled incorrectly as being lower when, in fact, this system is more secure.
Other considerations	No other considerations given.
Suggestions for improvement	The author suggests the assignment of passwords as an improvement. This is actually part of the design; it is not an improvement.
All relevant calculations	Calculations are not shown in the report.

Here are some things that could have been done to improve the report:

- **The Number of Possible Passwords**

  No improvement needed here.

- **The Probability of Users Forgetting Their Passwords**

  Calculate the probability of users forgetting their password using the data from the table on page 40 of the textbook. You may wish to report this as calculated in question 1 for the probability that at least 1 of the 10 000 users will forget a password on one day.

- **The Level of Security Offered**

  Calculate and list the probability of a password being guessed. Note that a smaller probability of guessing a password means a more secure system.

- **Other Considerations**

  The shortage of passwords for the longer time frame of 5 years should be noted. List any other considerations you may have.

- **Suggestions for Improvement**

  List any other improvements you may have.

- **All Relevant Calculations**

Show the calculations.

For the recommendations letter written on page 48 of the textbook, one concern that has not been mentioned is the complexity of entering a password like BLIO. Not only do you have to remember the four characters, but you have to remember how many times to press each key. Other possible suggestions would be that passwords need not be unique since the phone number uniquely indentifies the customer already. This would mean that there isn't a need for an ever-increasing number of passwords.

Coin Toss

Group	Results of 3 Coins Flipped 30 Times
1	HHH HTH THH TTH HTT THT TTT THT HHH THH TTH HTH THT TTT HHH HTT HTT THT HHT THH THT THT THH HHT THT THT THH TTH THT HHH
2	HHT TTT HHH HTT THT TTT HHH HTT TTT THT HHH HTH THT TTH HTH HHH HHH HHH THH HHT HTH THT TTH HTT THT THH HTT TTT HHH TTH
3	THH HTH HHH THH HTH THH HHH THT THT TTT HTH HTH THH HHH HHH HHH HTT THT HHH TTH HHH HTT HTT THT TTT THT HHH TTH HHH THH
4	HHH THH THT HTH HHH HTH THT TTH THH HTH HTT HHT THT HTH HTH THH HHH THT TTH THH TTH TTT THT HHT THT TTH THH THH HTT HTT
5	HHH HHT TTH HHT HHH THT HHH THT HTH HTT THT HTT THH TTT HTT HTH HTT HTT TTT HHH THH THT HHT THT HHH HTH HTT THT HHT HTT
6	HHH THT HTH TTH HTH HHT TTT HTH THT TTH HTH HHT TTH THH THH TTT THH THH HHT HTT THT TTT THH HHH HHH HTT THH TTH THT TTH
7	HTT HHH THT TTT HTH HTH TTT THT HHT HHH HHH HHH HHT THT THH HTT TTT HHT HTT HHH THH HHH TTH HTT HHT HTH HTH TTH TTT TTH
8	THH HHH THT HTT TTT THH HTH THT THH HHT HTT HHT HHH HTH HHT HTT TTH HTH HTT THH HHH TTH HHH THT TTH HTT HHT HTT HHH HTT
9	HHH THH HTH HHT HHH HTH HTH HTH HTT HHT HTT TTT HHH THT HTH HTH THT THT HHT TTT HTH HTH THH TTH HTT TTT HTT THT HHT HHH

10	HHT TTH HTH HTH HTH HTT TTT HTH TTH HHT TTH HTH HTH HTT HHH THH THH TTT HTH HTT THH HTT HHH HTH TTH TTH THH TTT HTH HTH
11	TTH TTH THH TTT HTH THH TTT THH HHT HTT THH THT HTH TTH THH HTH TTH TTH HTH THT TTT HTH TTH HTT TTH THH HTH TTT HHT HHH
12	TTT HHT TTT TTT THH TTT HTH THH HHT TTT HTH TTT HHH TTT TTH HHT HHT THH HHT HTT HTT HTH THH HTT HHH TTH THT TTT HHH HHT
13	HTT HHT HHT TTT HTT HHH THT HHT HTT THH THH HTT HTH HHH TTT TTH HHT TTH HTH TTT HHT THH TTH HHH HHT TTT HHH THH HTH HHH
14	HHT TTH THT HTH THH THH HTH HTH TTT HHH THH HHT TTT THH HHT HHT HHT TTH HTT HTH HHT THH HHH HTT HTH TTH THH THT HTH HHT
15	HHT HHH TTH HHH HTT THT TTH THT HTH HTT THH HHH HHT THT TTT HHH THH TTT TTT TTH HHT TTH THH THT TTT TTT HTH THH HHT TTT
16	HHH HHH THT THT HHT THH HHH TTT HHT HHH TTH HTT HHH HHH HTH HHH THT THT HHH HHT TTH HTT HTT HTT TTT THT TTH HTH TTT THH
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22	TTH TTH HHH TTT THT HHH HTH TTT TTT HHT HHT HTH HHT TTT HHH THH TTT HHH HHT THH THT TTT THH TTH HHT HTH HHT HTH TTT THH
23	HHH TTT HHT TTT HTH HHH THT THT HHT HHT HTH TTT HHH HHH THH HHH HHT HHT HTT TTH HHT THT HTT TTT HHH TTT HHT TTT TTT THH
24	HHT HHH HHT THH HHT HTH HTT THT TTT HHT TTT THH HHH THT THH TTT HHH HHH HTT HTT HTH HHH HTH HHT HHT TTT HTT TTT HHT HTH
25	HHT TTH TTT TTH HTT TTH TTT HTT HHH THT HHH TTH THT HHH THT HTT HTH THT THT THT HTH THT HTH HHT TTT TTT HHH HTT HTT HTH

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22	Image Club StudioGear/EyeWire Collection/Getty Images
23	Image Club StudioGear/EyeWire Collection/Getty Images
24	**top:** Image Club StudioGear/EyeWire Collection/Getty Images **bottom:** PhotoDisc Collection/Getty Images
26	PhotoDisc Collection/Getty Images
27	PhotoDisc Collection/Getty Images
28	EyeWire Collection/Getty Images
29	**collage:** PhotoDisc Collection/Getty Images
31	Image Club StudioGear/EyeWire Collection/Getty Images
34	Image Club StudioGear/EyeWire Collection/Getty Images
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